Supporting learning
Foundation Stage
Key Stage 1
Key Stage 2

# Stapleford Community Primary School

## A guide to helping your child with calculations

### Calculation Policy

**Aspire, Challenge, Discover**

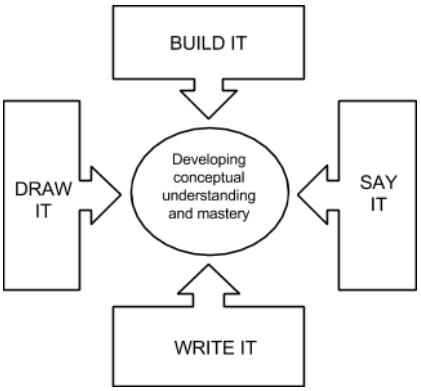
**Introduction**

This booklet explains the different methods of calculation which are taught at Stapleford Primary School and the order in which they are introduced. It has been written in accordance with the National Curriculum 2014 and supports the development of the three main aims of **Fluency, Reasoning** and **Problem Solving.** It is designed to give pupils a consistent and smooth progression of learning when using the four number operations. Our expectation is that children will leave primary school with a "toolkit" of calculation methods from which they can choose the best or most efficient method.

The following pages describe a combination of mental methods, methods which use a few jottings and formal written methods similar to those which many adults today learnt at school. There is a strong focus on understanding the maths behind these methods by ensuring that children have the opportunity to use them when solving real-life problems and challenges. In each case we have identified the age range when the method will be taught and consolidated. The teaching of later methods does not mean that children stop using earlier ones; they simply expand their "toolkit".

Mathematical understanding is developed through use of representations that are first of all **concrete** (e.g. Numicon, Dienes apparatus), and then **pictorial** (e.g. Array, place value counters) to then facilitate **abstract** working (e.g. Columnar addition, long multiplication). This policy gives examples of an appropriate progression of representations, and if at any point a pupil is struggling they should revert to familiar pictorial and/or concrete materials/ representations as appropriate.

As a school, in order to support the development of conceptual understanding and mastery, we use an approach known as **build it, draw it, say it, write it** which runs alongside and supports all aspects of the Primary Maths Curriculum.




**Aims of the Calculation Policy**

- To support greater consistency in the teaching of written calculations across the school.
- To strengthen continuity and progression in children's understanding of the development of written calculations.
- To form a core set of methods which every children will experience and build upon.
- To build on models and images introduced to promote conceptual understanding.
- To provide reference and guidance on the teaching of calculation skills for teaching staff, teaching assistants and parents.

# Contents

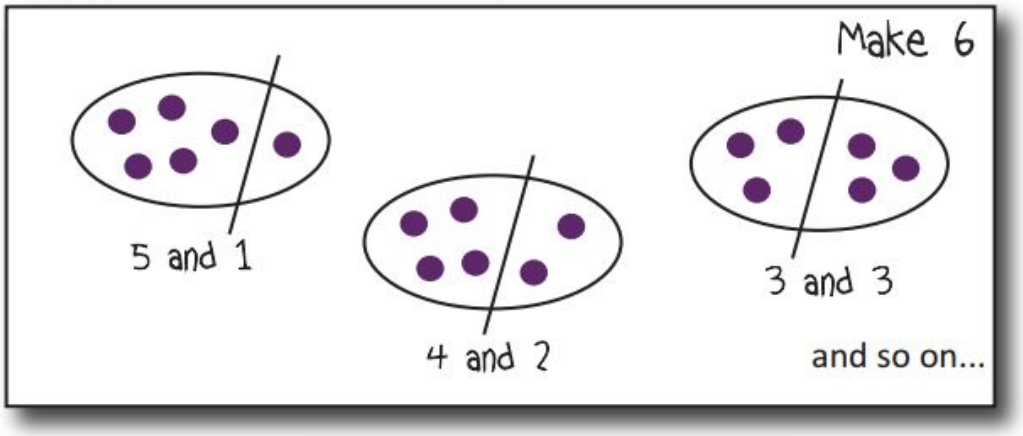
# Addition

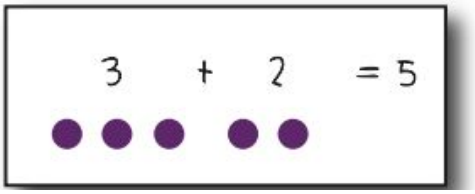
| | Pupils should be taught to |
|---|---|
| **Foundation Stage** | • one more and one less<br>•investigating ways of making numbers within 10 |
| **Year 1** | • read, write and interpret mathematical statements involving addition (+) and equals (=) signs<br>• represent and use number bonds and related facts within 20<br>• add one-digit and two-digit numbers to 20, including zero<br>• solve one-step problems that involve addition, using concrete objects and pictorial representations, and missing number problems such as 7 = _ + 9 |
| **Year 2** | • solve problems with addition using concrete objects and pictorial representations, including those involving numbers, quantities and measures<br>• apply their increasing knowledge of mental and written methods<br>• recall and use addition facts to 20 fluently, and derive and use related facts up to 100<br>• add numbers using concrete objects, pictorial representations, and mentally, including:<br>-a two-digit number and ones<br>-a two-digit number and tens<br>- two two-digit numbers<br>-adding three one-digit numbers<br>e.g. 6 + 7 + 4<br>• show that addition of two numbers can be done in any order (commutative)<br>• recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems<br>• to record addition in columns as this supports place value and prepares for formal written methods with larger numbers |
| **Year 3** | • add numbers mentally, including:<br>-a three-digit number and ones<br>-a three-digit number and tens<br>-a three-digit number and hundreds<br>• add numbers with up to three digits, using formal written methods of columnar addition<br>• estimate the answer to a calculation and use inverse operations to check answers<br>• solve problems, including missing number problems, using number facts, place value, and more complex addition |
| **Year 4** | • add numbers with up to 4 digits using the formal written methods of columnar addition (expanded) where appropriate<br>• estimate and use inverse operations to check answers to a calculation e.g. estimate 8203 + 499 by 8200 + 500 = 8700. Check the inverse = 8702<br>• solve addition two-step problems in contexts, deciding which operations and methods to use and why |
| **Year 5** | • add whole numbers with more than 4 digits, including using formal written methods. (columnar addition)<br>• add numbers mentally with increasingly large numbers<br>• use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy.<br>• solve multi-step problems in contexts, deciding which operations and methods to use and why |
| **Year 6** | • add whole numbers with more than 4 digits, using different strategies, including the formal written methods of columnar addition<br>• perform mental calculations, including with mixed operations and large numbers<br>• use their knowledge of the order of operations to carry out calculations involving the four operations<br>• solve addition multi-step problems in contexts, deciding which operations and methods to use and why<br>• use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy |
| **Vocabulary** | add, more, plus, make, altogether, equals, most, count on, numberline, tens, units,<br>addition, number sentence , tens, units, partition, addition<br>hundreds, boundary,<br>thousands, hundreds, digits, inverse.<br>decimal place, decimal point, tenths, hundredths, thousandths.<br>integer |

# Addition

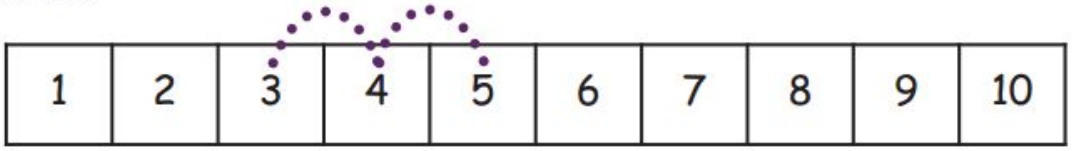
**Stage 1 (Foundation Stage - Year 1)**

Children first use a range of objects and pictures to explain addition as combining two sets of objects. Beginning with the objects themselves, children learn to do addition by drawing pictures of the objects and later representing the objects by symbols such as dots or tally marks.

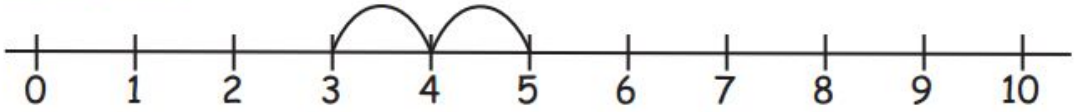



Once this process is secure, children begin to use a **number track** to help them count on...




...or a **numberline** marked in steps of 1.

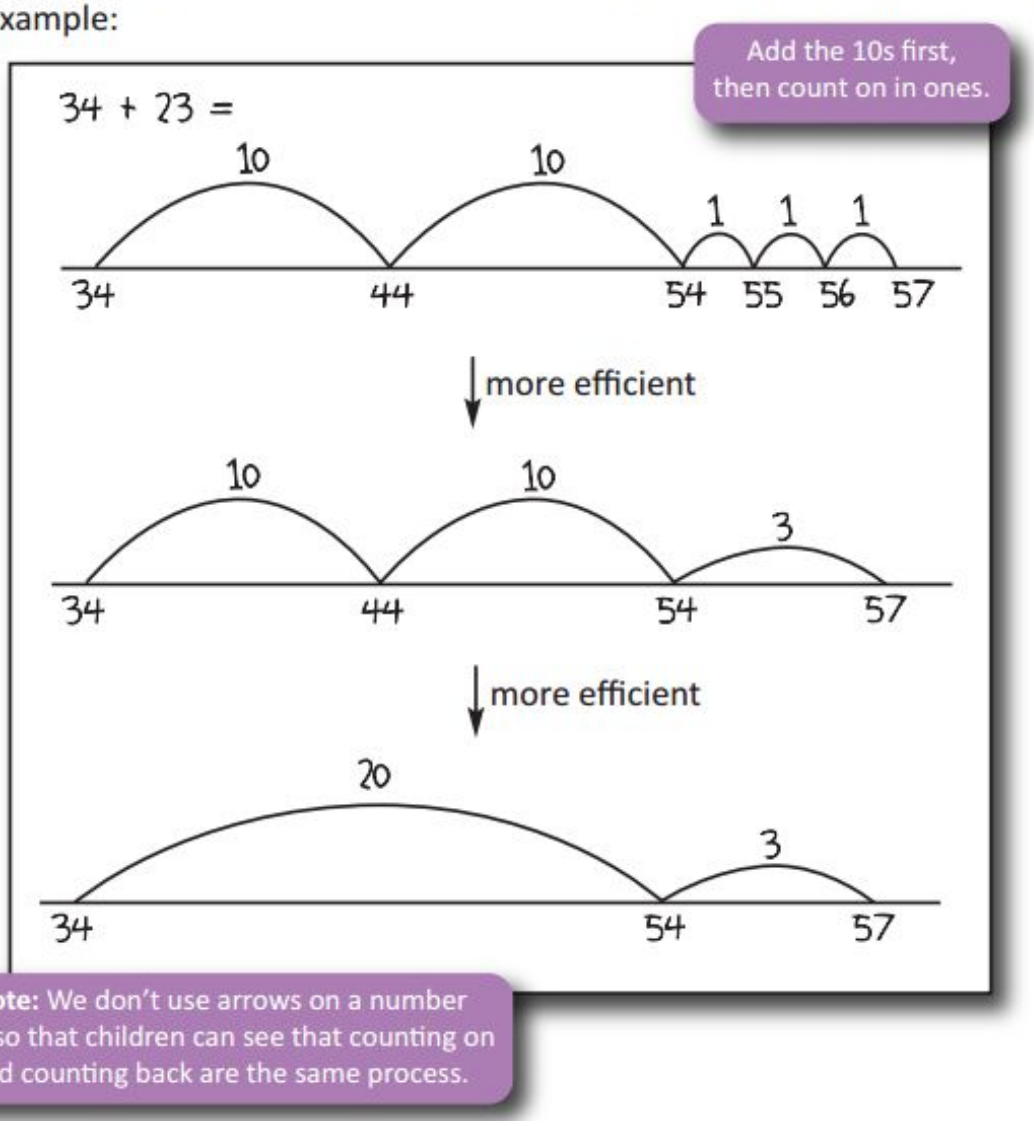



**Note:** Labelled numberlines like this should always start at 0 (zero).

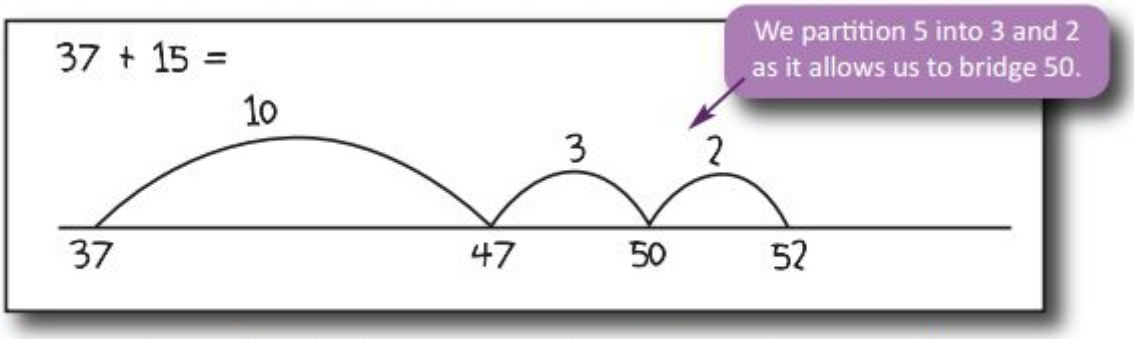
During Year 1 children will also use **hundred squares** and **bead strings** to help them calculate with numbers up to 20.

**Stage 2** (around Year 2)

Empty numberlines are used at stage 2 to count up, becoming increasingly efficient as children become more confident in choosing the jumps that they use. For example:



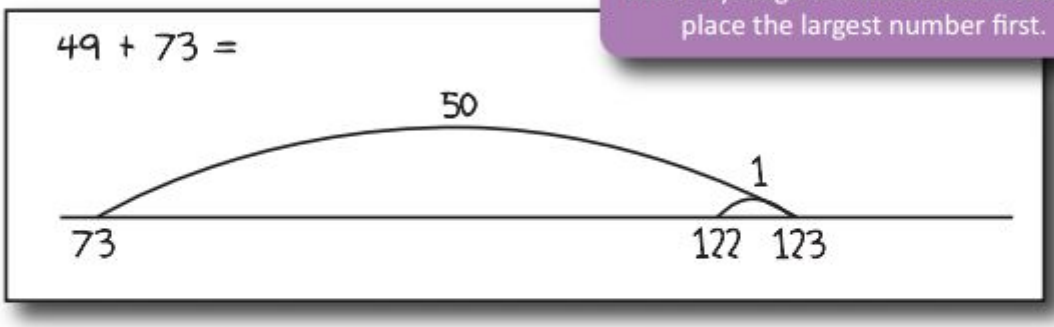

**Bridging** across 10 is another way of making the process more efficient.




Children will also use **hundred squares** and **bead strings** to support their addition of numbers to 100.

**Stage 3** (Year 2-3)

Children use the numberline to support addition for increasingly large numbers. In addition, they learn to add numbers such as 49 by adding 50 and taking away 1.



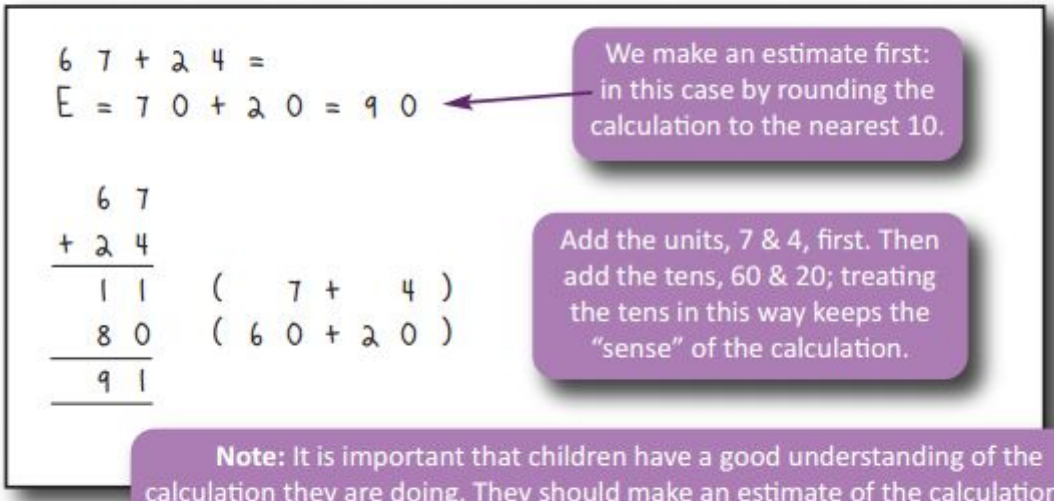

**Note:** By stage 3 children automatically place the largest number first.

Pencil and paper methods without a number line begin at this stage by **partitioning**. For example:


**Partitioning** splits each number into the tens and units - we always add the tens first, then the units, as this is how we would do the calculation mentally.

**Stage 4** (end Year 3 - Year 4)

Our first "written method" is an expanded version. Because this is a written method and not a mental one, and so that children are ready to "carry", we **begin from the right** with the **least significant digits**.



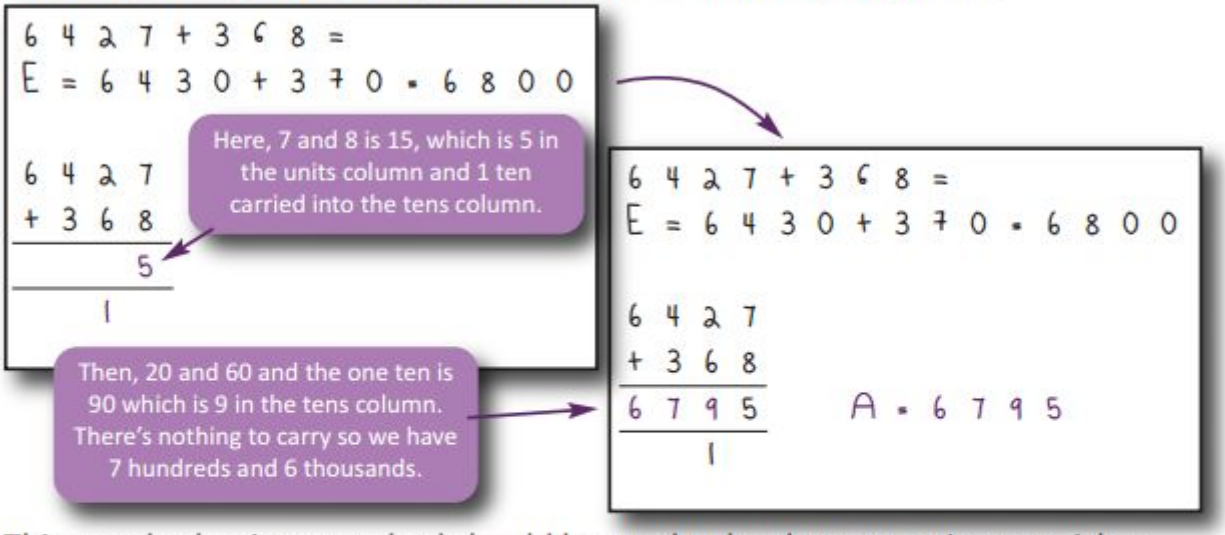

We make an estimate first: in this case by rounding the calculation to the nearest 10.

Add the units, 7 & 4, first. Then add the tens, 60 & 20; treating the tens in this way keeps the "sense" of the calculation.

**Note:** It is important that children have a good understanding of the calculation they are doing. They should make an estimate of the calculation first - in this way they should be able to spot any calculation errors. This is helped by making sure that calculations have a context in which they make sense.

**Stage 5** (Year 4 - Year 6)

The common standard written method for addition is used at stage 5, making sure that children have made an estimate first to pick up any errors.




This standard written method should be used only when an easier or quicker method is not available. As children become more confident they will be able to use the method to:

- add more than two numbers with different numbers of digits
- add money, lining up the decimal points, and dealing with mixed amounts, eg £3.59 + 78p
- add two or more decimal fractions with up to two decimal places
- add quantities in mixed units, eg 3.2m + 280cm

Billy uses a pedometer to measure how far he walks in three days. On Monday he walked 4.6km, on Tuesday he walked 5km and on Wednesday he walked 780m. How far did he walk altogether?

4.6 km + 5 km + 780 m =
E = 5 km + 5 km + 1 km = 11 km

4.60
5.00
0.78
10.38
1
A = 10.38 km

Children will also use practical resources and diagrams to help them add improper fractions and mixed numbers.

# Subtraction

| | Pupils should be taught to |
|---|---|
| **Foundation Stage** | •counting forwards and backwards in 1s<br>• making number facts within 1 |
| **Year 1** | • read, write and interpret mathematical statements involving, subtraction (–) and equals (=) signs<br>• represent and use number bonds and related subtraction facts within 20<br>• subtract one-digit and two-digit numbers to 20, including zero<br>• solve one-step problems that involve subtraction, using concrete objects and pictorial representations, and missing number problems such as 7 = _ – 9 |
| **Year 2** | • solve problems with subtraction using concrete objects and pictorial representations, including those involving numbers, quantities and measures<br>• apply their increasing knowledge of mental and written methods<br>• recall and use subtraction facts to 20 fluently, and derive and use related facts up to 100<br>• subtract numbers using concrete objects, pictorial representations, and mentally, including:<br>-a two-digit number and ones<br>-a two-digit number and tens<br>e.g. 87 – 30 = 57<br>- two two-digit numbers<br>-subtracting three one-digit numbers<br>• Understanding that subtraction of one number from another cannot be done in any order<br>• recognise and use the inverse relationship between addition and subtraction and use this to check calculations and solve missing number problems<br>• to record subtraction in columns as this supports place value and prepares for formal written methods with larger numbers |
| **Year 3** | • subtract numbers mentally, including:<br>-a three-digit number and ones<br>-a three-digit number and tens<br>-a three-digit number and hundreds<br>• subtract numbers with up to three digits, using formal written methods of columnar subtraction<br>• estimate the answer to a calculation and use inverse operations to check answers<br>• solve problems, including missing number problems, using number facts, place value, and more complex subtraction |
| **Year 4** | • subtract numbers with up to 4 digits using the formal written methods of columnar subtraction (expanded) where appropriate<br>• estimate and use inverse operations to check answers to a calculation<br>• solve subtraction two-step problems in contexts, deciding which operations and methods to use and why |
| **Year 5** | • subtract whole numbers with more than 4 digits, including using formal written methods. (columnar subtraction)<br>• subtract numbers mentally with increasingly large numbers<br>• use rounding to check answers to calculations and determine, in the context of a problem, levels of accuracy<br>• solve multi-step problems in contexts, deciding which operations and methods to use and why |
| **Year 6** | • subtract whole numbers with more than 4 digits, using different strategies, including the formal written methods of columnar subtraction<br>• perform mental calculations, including with mixed operations and large numbers<br>• use their knowledge of the order of operations to carry out calculations involving the four operations<br>• solve subtraction multi-step problems in contexts, deciding which operations and methods to use and why<br>• use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy |
| **Vocabulary** | take away, less, minus, subtract, how many more, how many fewer/less than, most, least, how many left, partition, tens, units,<br>difference between, count back, how many left, count on, digit<br>distance between,<br>exchange, decrease, hundreds, value, digit<br>inverse, tenths, hundredths, decimal place, decimal |

# Subtraction

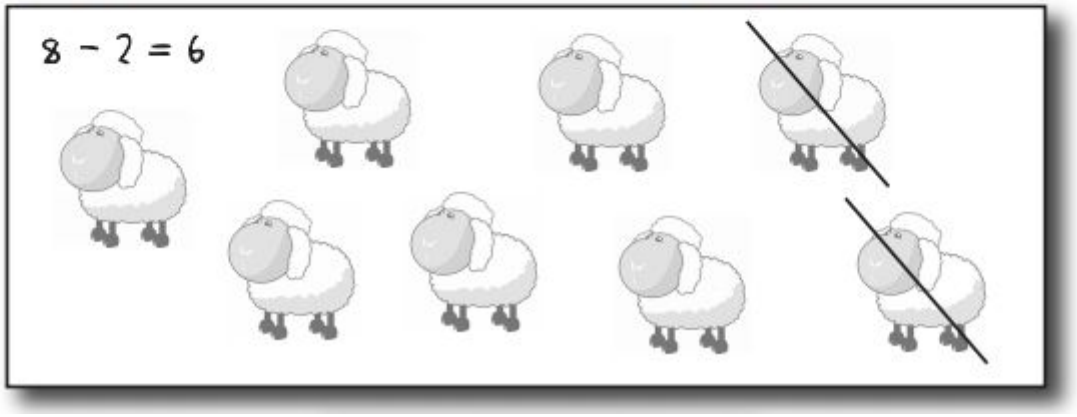
Subtraction can be described in three ways:

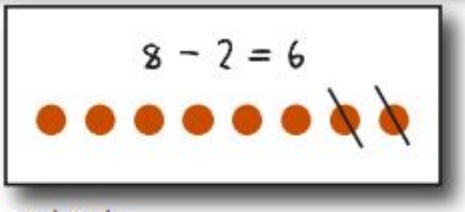
- taking away
- counting back
- finding the **difference** (counting on)

It is important that children understand the relationship between these three different interpretations of subtraction.

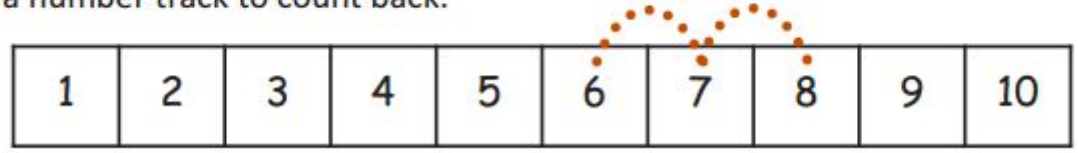
**Stage 1** (Foundation Stage - Year 1)

Real objects, pictures and symbols come first. Taking away is easiest.

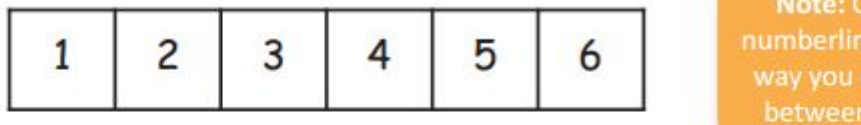



Use a number track to count back:




Or find the difference by comparing two number tracks:



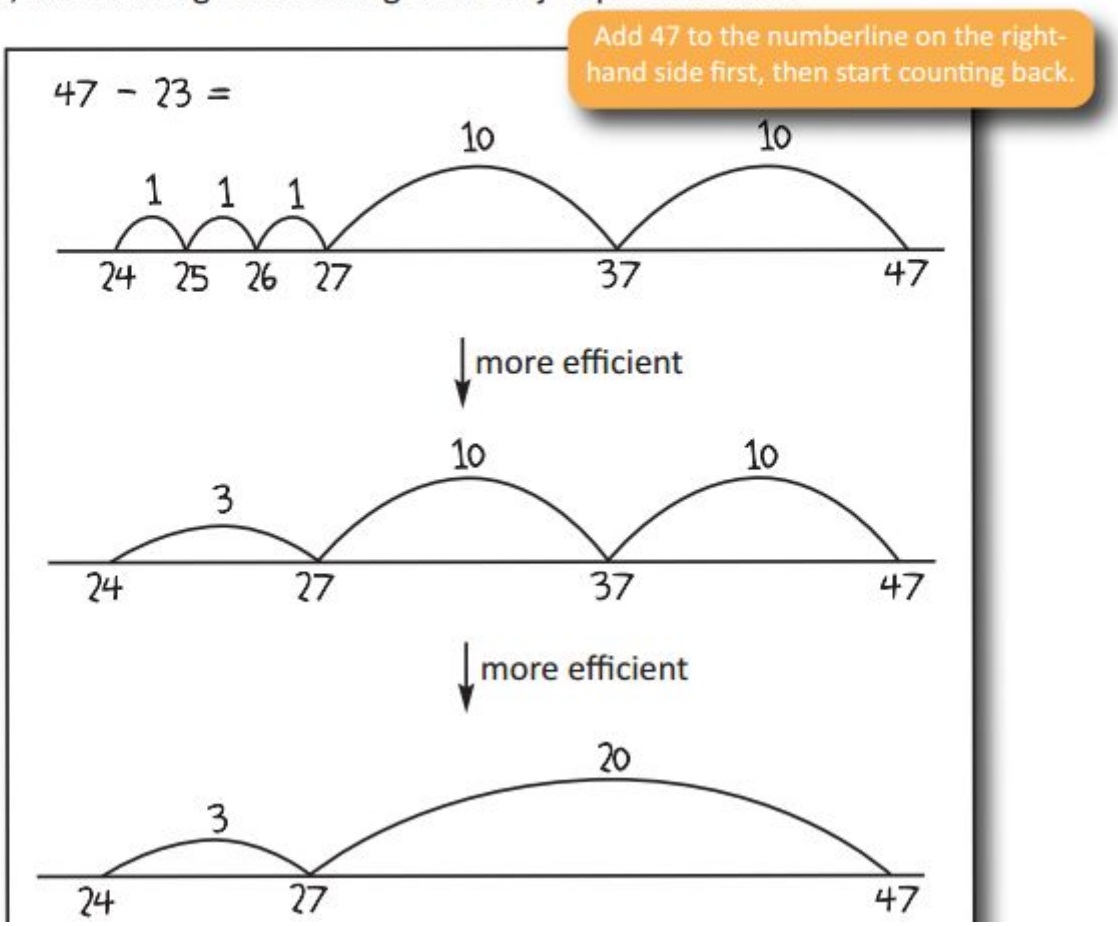


**Note:** Counting on with a numberline works too, but this way you can see a **difference** between the two numbers.

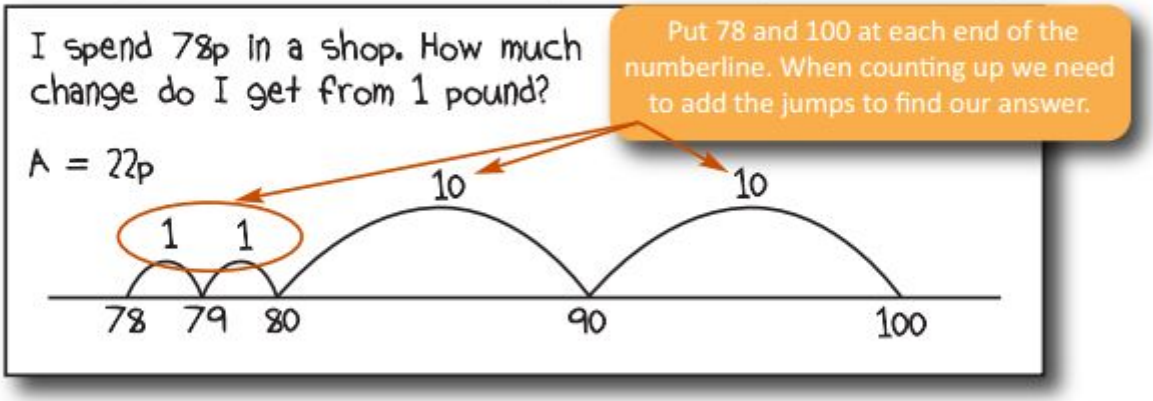
In Year 1, children will use number lines, bead strings and hundred squares to calculate numbers up to 20.

**Stage 2** (around Year 2)

Using an empty numberline we count backwards from the right. As with addition, the challenge is choosing efficient jumps backwards.



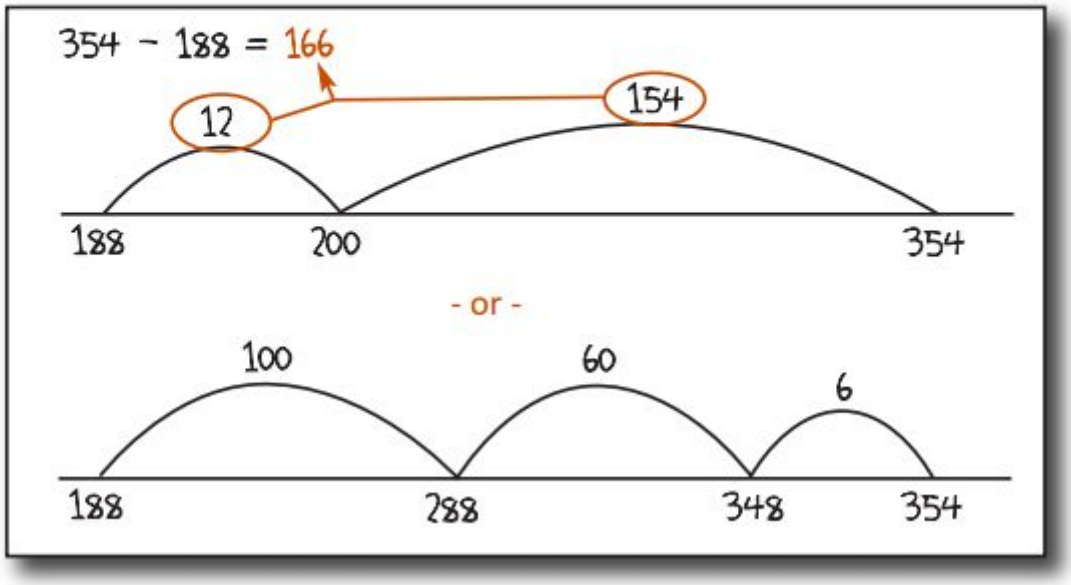

If the numbers involved in the calculation are close together or close to multiples of 10 or 100 etc, it can be easier to count on. In fact, many children find counting on more straightforward. A common example is counting on to find change.



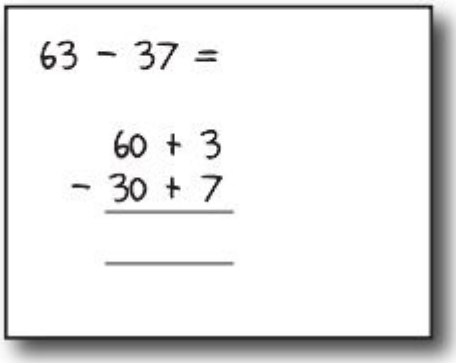

Children will continue to use beadstrings and hundred squares to support their subtraction of numbers to 100.

**Stage 3** (around Year 3)

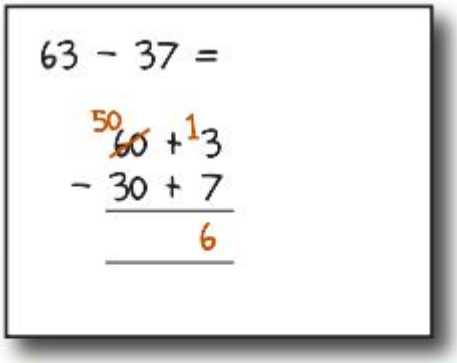
At this stage, children are using numberlines with much greater skill; they choose the best methods and the best jumps for subtraction calculations less than 1000.

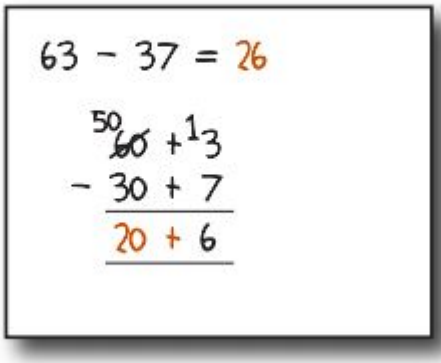



Pencil and paper methods without using a numberline begin with a simple, expanded method which makes explicit the need to move tens into the units column to carry out a subtraction like 63 - 37. As with addition, we must **start at the right** with the **least significant digits**.




Set out the calculation as shown here on the left, **partitioning** the tens and units. In order to "take 7 from 3" we need to move ten to the units column. This is called **exchanging**. At this point a common mistake is to reverse the calculation and take 3 from 7; remember that we are taking 37 from 63.




Once we have moved 10 we can take 7 from 13, leaving 6. Then we take 30 from the new value of 50.

**Stage 4** (Year 4 - Year 6)

This is the final stage for subtraction. We contract our expanded written method into the standard method: **decomposition**. Again, start on the right.

754 - 286 =
E = 750 - 300 = 450

754
- 286
———

In this calculation, first move a ten into the units column. A common mistake is to reverse the calculation, taking 4 from 6. Repeat, moving a hundred into the tens column.

754 - 286 =
E = 750 - 300 = 450

7 ~~5~~ ⁴14
- 286
———
8

754 - 286 =
E = 750 - 300 = 450

6 ~~7~~ ¹⁴~~5~~ 14
- 286
———
68

This method is highly error prone. It is essential that children check their answer against their estimate.

754 - 286 =
E = 750 - 300 = 450

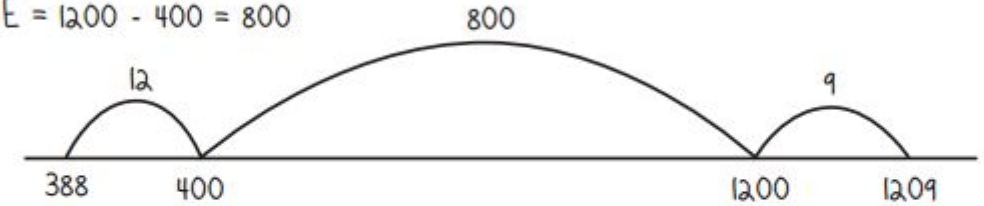
6 ~~7~~ ¹⁴~~5~~ 14
- 286
———
468

A = 468

Numberlines remain easier and more reliable in some cases and children by now should be confident with this process.

1209 - 388 =
E = 1200 - 400 = 800

A = 821




As with addition, children should be able to calculate with:

- decimal fractions with different numbers of digits
- different units, eg 5.67kg - 870g
- numbers with 5 or 6 digits (Year 5)
- improper fractions and mixed numbers

# Multiplication

| | Pupils should be taught to |
|---|---|
| **Foundation Stage** | • Count forwards and backwards in 5s and 10s<br>• Investigate doubling |
| **Year 1** | • count in multiples of twos, fives and tens<br>• double and halve numbers to 20, e.g. double 3 is 6, half of 10 is 5<br>• make connections between arrays and number patterns<br>• solve simple one-step problems involving multiplication, calculating the answer using concrete objects, pictorial representations and arrays |
| **Year 2** | • count in multiples of twos, threes, fives and tens and recite the multiplication tables<br>• calculate mathematical statements for multiplication within multiplication tables and write them using the multiplication (x) and equals (=) signs<br>• count in multiples of twos, threes, fives and tens and recite the multiplication tables<br>• show that multiplication of two numbers can be done in any order (commutative)<br>e.g. 4 x 3 = 12 3 x 4 = 12<br>• solve problems involving multiplication and using materials, arrays, repeated addition, mental methods and multiplication facts, including problems in contexts<br>E.g. I have 3 bags of apples. Each bag contains 8 apples, how many apples altogether? |
| **Year 3** | • count from 0 in multiples of 4, 8, 50 and 100<br>• recall and use multiplication facts for the 3, 4 and 8 multiplication tables<br>• write and calculate mathematical statements for multiplication using the multiplication tables that they know, including for two-digit numbers times one-digit numbers, using mental and progressing to formal written methods<br>• using commutativity and associativity (for example 4 x 12 x 5 = 4 x 5 x 12 = 20 x 12 = 240)<br>• solve problems, including missing number problems, involving multiplication, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects |
| **Year 4** | • Count in multiples of 6, 7, 9, 25, and 1000<br>• recall multiplication and division facts for multiplication tables up to 12 × 12<br>• use place value, known and derived facts to multiply mentally, including: multiplying by 0 and 1; dividing by 1; multiplying together three numbers<br>• recognise and use factor pairs and commutativity in mental calculations<br>• multiply two-digit and three-digit numbers by a one-digit number using formal written layout<br>• solve problems involving multiplying and adding, including using the distributive law to multiply two digit numbers by one digit, integer scaling problems and harder correspondence problems such as n objects are connected to m objects |
| **Year 5** | • continue to use all multiplication tables to 12 x 12 in order to maintain their fluency<br>• identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers<br>• know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers<br>• establish whether a number up to 100 is prime and recall prime numbers up to 19<br>• multiply numbers up to 4 digits by a one- or two-digit number using a formal written method, including long multiplication for two-digit numbers<br>• multiply numbers mentally drawing upon known facts<br>• multiply whole numbers and those involving decimals by 10, 100 and 1000<br>• recognise and use square numbers and cube numbers, and the notation for squared ($^2$) and cubed ($^3$)<br>• solve problems involving multiplication including using their knowledge of factors and multiples, squares and cubes<br>• solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign<br>• solve problems involving multiplication, including scaling by simple fractions and problems involving simple rates |
| **Year 6** | • continue to use all multiplication tables to 12 x 12 in order to maintain their fluency<br>• multiply multi-digit numbers up to 4 digits by a two-digit whole number using the formal written method of long multiplication<br>• perform mental calculations, including with mixed operations and large numbers<br>• identify common factors, common multiples and prime numbers<br>•use their knowledge of the order of operations to carry out calculations involving the four operations<br>• solve problems involving addition, subtraction, multiplication and division<br>• use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy |
| **Vocabulary** | times, lots of times, array, altogether, multiply, count, tens, units<br>groups of, lots of, multiplied by, repeated addition, column, row,<br>commutative, sets of, equal groups, product<br>partition, grid method, multiple, product, inverse<br>square, factor, integer, decimal, short/long multiplication, |

# Multiplication

**Stage 1** (Foundation Stage - Year 1)

Early work on multiplication involves counting on in steps of 2 initially, then in steps of 5 and 10. The concept of multiplication at this stage is **entirely** practical - it involves exploring real-life examples of equal sets or groups.

I have 4 pairs of socks. How many socks are there?

Just as with addition and subtraction, children can begin to substitute symbols for real objects.

I have 3 boxes of 6 eggs. How many eggs?

Representing numbers in this way, i.e. in a grid, is called an **array**. In this example you can also see that the array shows that 6 is 3 lots of 2 and also 2 lots of 3.

**Stage 2/3** (Year 2)

At stages 2 and 3 we represent multiplication as repeated addition. So, the following expressions all show the same calculation:

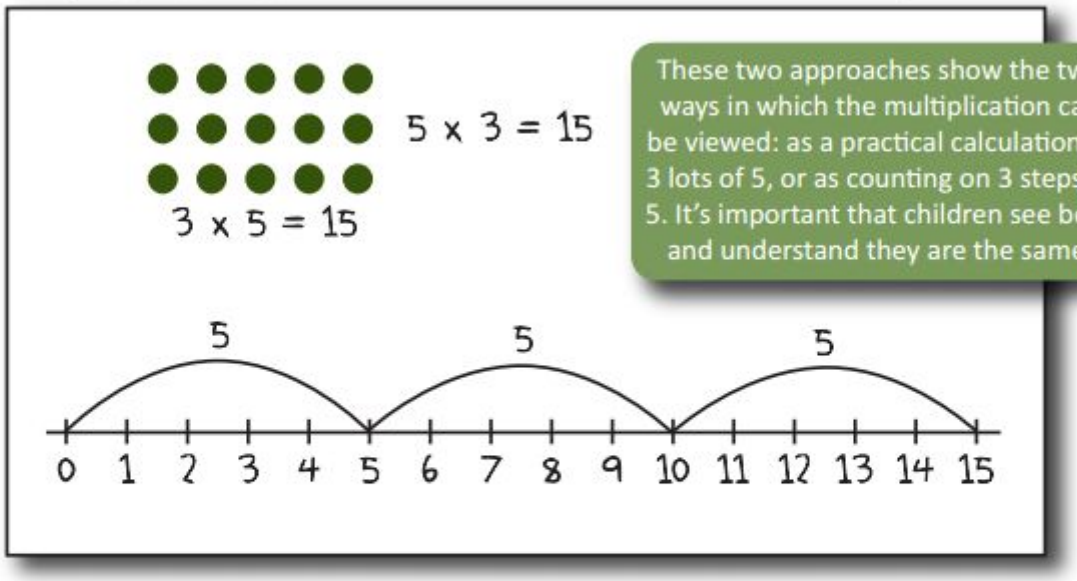
3 times 5 5 + 5 + 5 3 lots of 5 3 x 5

Multiplication (like addition) is commutative: that is, 3 x 5 is the same as 5 x 3. Children use this fact, with repeated addition, to calculate simple multiplications.

4 x 7
= 7 + 7 + 7 + 7
=28

7 x 4
= 4 + 4 + 4 + 4 + 4 + 4 + 4
= 28

Using a grid (**array**) or a numberline, we can calculate a multiplication:



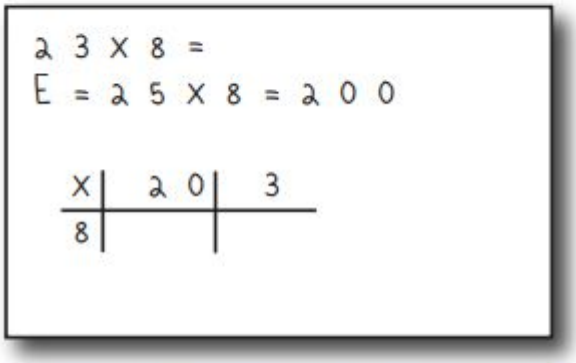

These two approaches show the two ways in which the multiplication can be viewed: as a practical calculation of 3 lots of 5, or as counting on 3 steps of 5. It's important that children see both and understand they are the same.

Both of these methods are used throughout stages 2 and 3 and are taught alongside the relevant tables in the following order:

- 2, 5 & 10 times tables (Year 2)
- 3, 4 & 8 times tables (Year 3)
- 6, 7, 9, 11 & 12 times tables (Year 4)

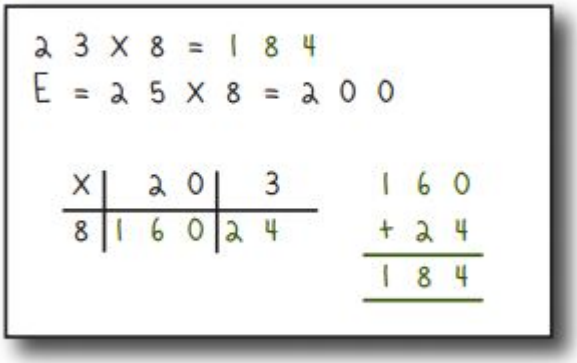
**Stage 4** (Year 3 and 4)

This stage introduces the '**grid method**' for multiplication. We begin with a straightforward calculation with a two-digit number (TU) multiplied by a single-digit number (U). Children will also use the grid method for three-digit numbers (HTU) multiplied by single-digit (U) numbers.




We complete an estimate first so that we can check our answer. Then we partition the two-digit number into its tens (20) and units (3). Set the question out in a grid as shown.

Now calculate 8 x 20 and place the answer in the grid, following this with 8 x 3. Add the two answers together the complete the calculation and check with the estimate.



By the end of stage 4, children will be able to use a formal written method of calculation for two-digit (TU) and three-digit (HTU) multiplied by a single digit. This is taught alongside the grid method which most children find easier to understand.

3 5 0 x 7 = 2 4 5 0

| | | | |
|---|---|---|---|
| | 3 | 5 | 0 |
| x | | | 7 |
| 2 | 4 | 5 | 0 |
| 2 | 3 | | |

**Stage 5** (Year 5 - Year 6)

Stage 5 builds on stage 4 by extending the grid method to a range of other possible calculations.

- ThHTU x U (eg 4346 x 8)
- TU x TU (eg 72 x 38) & HTU x TU (eg 372 x 24 - example 1 below)
- U.t x U (eg 4.9 x 3) & U.th x U (eg 6.73 x 7 - example 2 below)

In **example 1** there are two rows in the grid - one for the tens and one for the units.

3 7 2 X 2 4 =
E = 4 0 0 X 2 5 = 1 0, 0 0 0

| X | 3 0 0 | 7 0 | 2 |
|---|---|---|---|
| 2 0 | 6 0 0 0 | 1 4 0 0 | 4 0 |
| 4 | 1 2 0 0 | 2 8 0 | 8 |

A = 8, 9 2 8

| | | | | |
|---|---|---|---|---|
| | 6 | 0 | 0 | 0 |
| | 1 | 2 | 0 | 0 |
| | 1 | 4 | 0 | 0 |
| | | 2 | 8 | 0 |
| | | | 4 | 0 |
| + | | | | 8 |
| | 8 | 9 | 2 | 8 |
| | | 1 | | |

6.7 3 X 7 =
E = 7 X 7 = 4 9

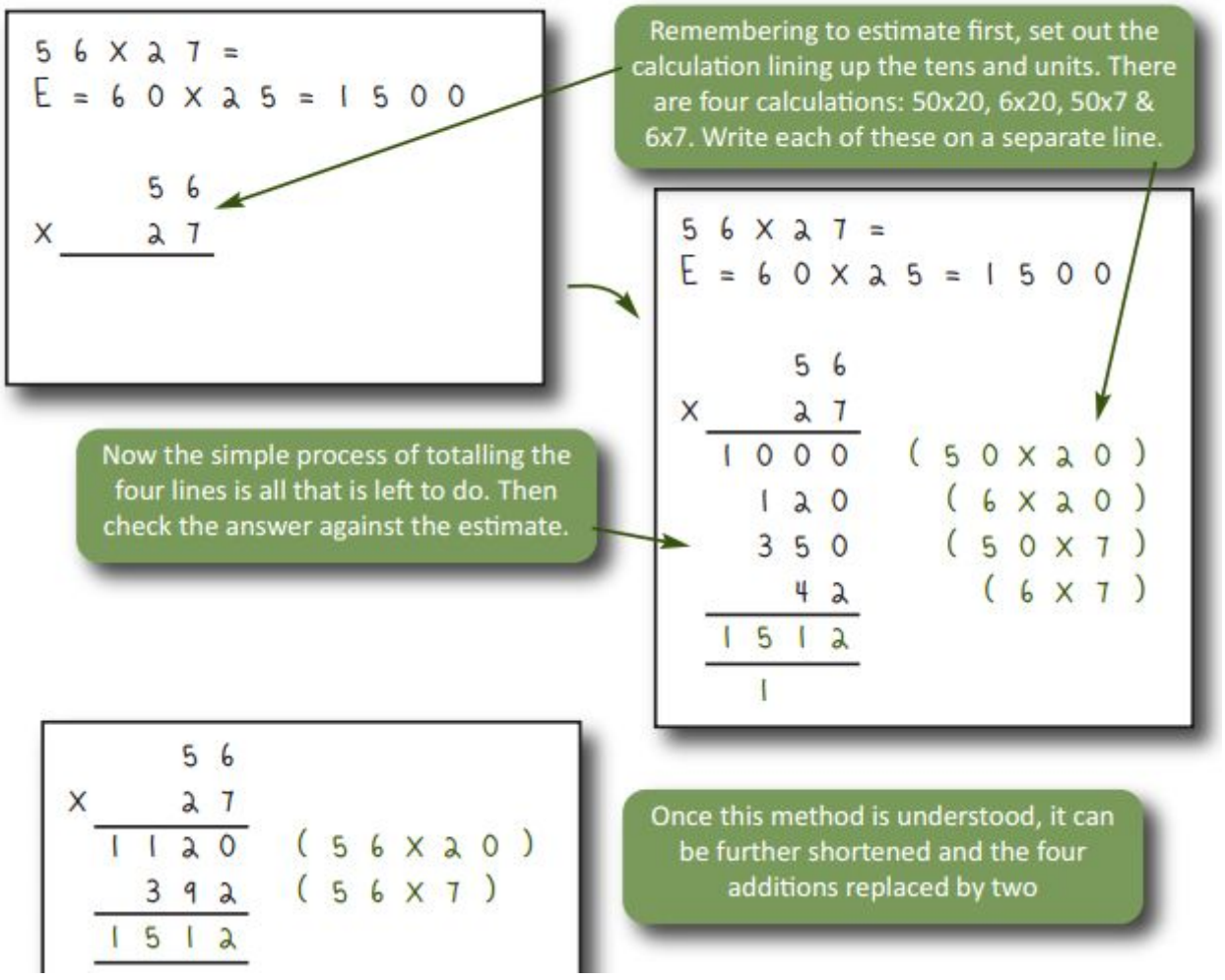
**Example 2** requires a good understanding of decimals.

| X | 6 | 0.7 | 0.0 3 |
|---|---|---|---|
| 7 | 4 2 | 4.9 | 0.2 1 |

A = 4 7.1 1

| | | | | |
|---|---|---|---|---|
| | | 4 | 2 | |
| | | | 4 | 9 |
| + | | | 0 | 2 1 |
| | 4 | 7 | 1 | 1 |
| | | 1 | | |

The final stage for this operation is the standard written method of long multiplication. It is easy to see how this method develops from the grid method as the processes are the same, with each section of the grid written in a column.




Even at this stage, many children prefer the visual nature of the grid method.

A school trip to Wimpole Hall costs £4.63 for each child. 23 children go on the trip. How much does it cost altogether?

E = £5 X 23 = £115

| X | 4 | 0.6 | 0.03 |
|---|---|---|---|
| 20 | 80 | 12 | 0.6 |
| 3 | 12 | 1.8 | 0.09 |

A - £106.49

80
12
12
1.8
0.6
+ 0.09
106.49
1

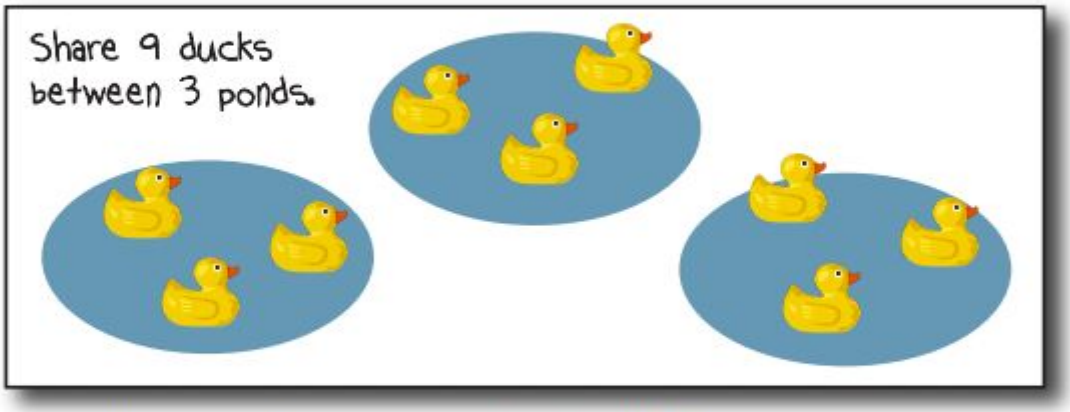
# Division

| | Pupils should be taught to |
|---|---|
| **Foundation Stage** | • Investigating halving |
| **Year 1** | • solve one-step problems involving division, by calculating the answer using concrete objects, pictorial representations and arrays with the support of the teacher<br>• double and halve numbers to 20<br>e.g. double 8 is 16, half of 20 is 10<br>• make connections between arrays, number patterns and counting backwards in twos, fives and tens |
| **Year 2** | • recall and use the division facts for 2, 5 and 10 multiplication tables, including recognising odd and even numbers.<br>• calculate division statements and write them using the division (x) and equals (=) signs<br>• show that division of one number by another is NOT commutative<br>• solve problems involving division using materials, arrays, repeated subtraction, mental methods and division facts, including problems in contexts<br>• Pupils work with a range of materials and contexts in which division relates to grouping and sharing discrete and continuous quantities. They begin to relate these to fractions and measures (40 divided by 2 is 20 and 20 is half of 40)<br>• They use commutativity and inverse relations to develop multiplicative reasoning. (for example 4 x 5 = 20, 20 divided by 5 = 4) |
| **Year 3** | • recall and use division facts for the 3, 4 and 8 multiplication tables<br>• write and calculate mathematical statements for division using the multiplication tables that they know, using efficient mental methods, e.g. using 3 x 2 = 6, 6 ÷ 3 = 2 to derive related facts of 30 x 2 = 60, 60 ÷ 3 = 20 and 20 = 60 ÷ 3 and progressing to formal written methods<br>• solve problems, including missing number problems, involving multiplication and division, including positive integer scaling problems and correspondence problems in which n objects are connected to m objects |
| **Year 4** | • recall division facts for multiplication tables up to 12 × 12<br>• use place value, known and derived facts to multiply and divide mentally, including: dividing by 1;<br>• recognise and use factor pairs and commutativity in mental calculations<br>• practise mental methods and extend this to three-digit numbers to derive facts, (for example 600 ÷ 3 = 200 can be derived from 2 x 3 = 6)<br>• Use efficient written method for division with exact answers when dividing by a one-digit number |
| **Year 5** | • know and use the vocabulary of prime numbers, prime factors and composite (non-prime) numbers<br>• establish whether a number up to 100 is prime and recall prime numbers up to 19<br>• divide numbers mentally drawing upon known facts<br>• divide numbers up to 4 digits by a one-digit number using the formal written method of short division and interpret remainders appropriately for the context<br>• divide whole numbers and those involving decimals by 10, 100 and 1000<br>• solve problems division including using their knowledge of factors and multiples, squares and cubes<br>• solve problems involving addition, subtraction, multiplication and division and a combination of these, including understanding the meaning of the equals sign<br>• solve problems involving division, including scaling by simple fractions and problems involving simple rates.<br>• interpret non-integer answers to division by expressing results in different ways according to the context, including with remainders, as fractions, as decimals or by rounding |
| **Year 6** | • divide numbers up to 4 digits by a two-digit whole number using the formal written method of long division, and interpret remainders as whole number remainders, fractions, or by rounding, as appropriate for the context<br>• divide numbers up to 4 digits by a two-digit number using the formal written method of short division where appropriate, interpreting remainders according to the context<br>• perform mental calculations, including with mixed operations and large numbers<br>• use their knowledge of the order of operations to carry out calculations involving the four operations<br>• solve problems involving addition, subtraction, multiplication and division<br>• use estimation to check answers to calculations and determine, in the context of a problem, an appropriate degree of accuracy |
| **Vocabulary** | share, share equally, groups of, lots of, array, divide, divided by<br>divided into, division, grouping, number line<br>left over,<br>inverse, short division, remainder, multiple<br>divisible by, factor<br>quotient, prime number, prime factors, composite number (non-prime) |

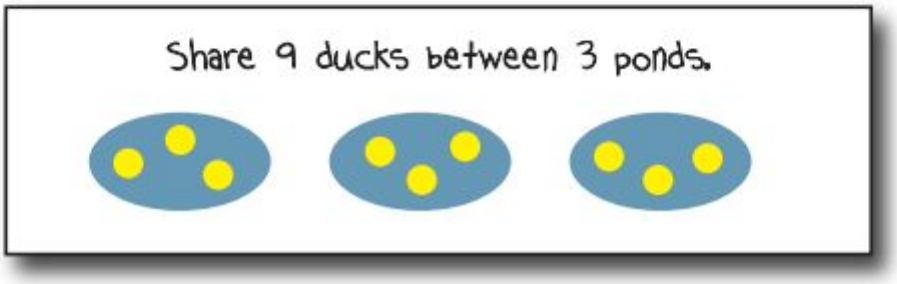
# Division

**Stage 1** (Foundation Stage - Year 1)

Early division involves sharing equally in practical and real-life contexts.



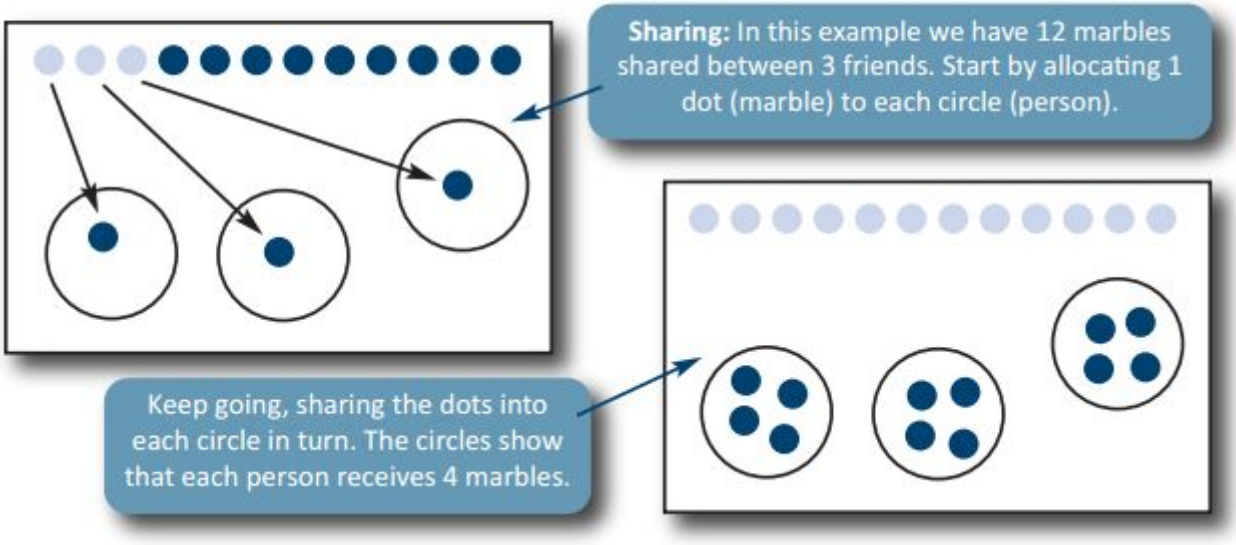

The same problem can be represented with symbols:

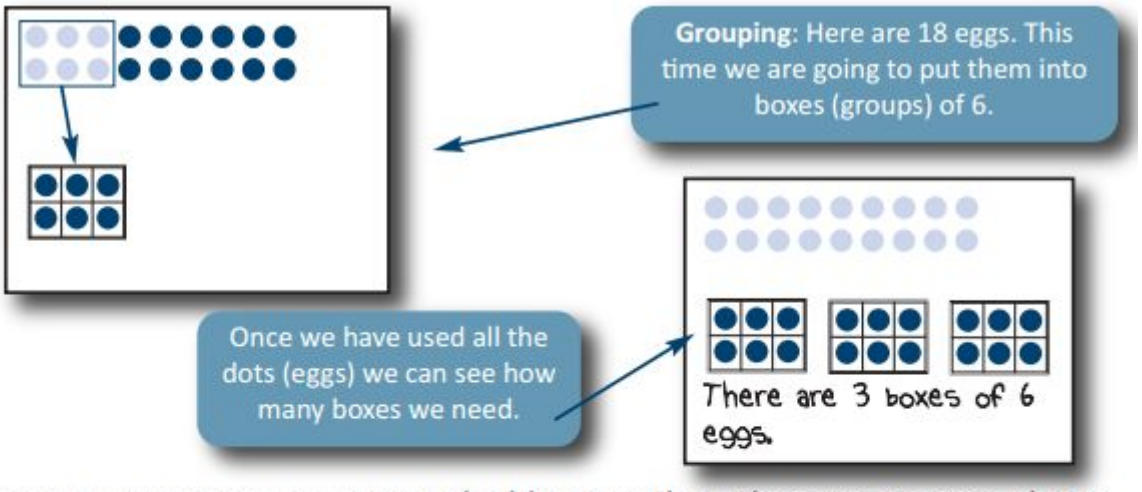



**Stage 2** (around Year 2)

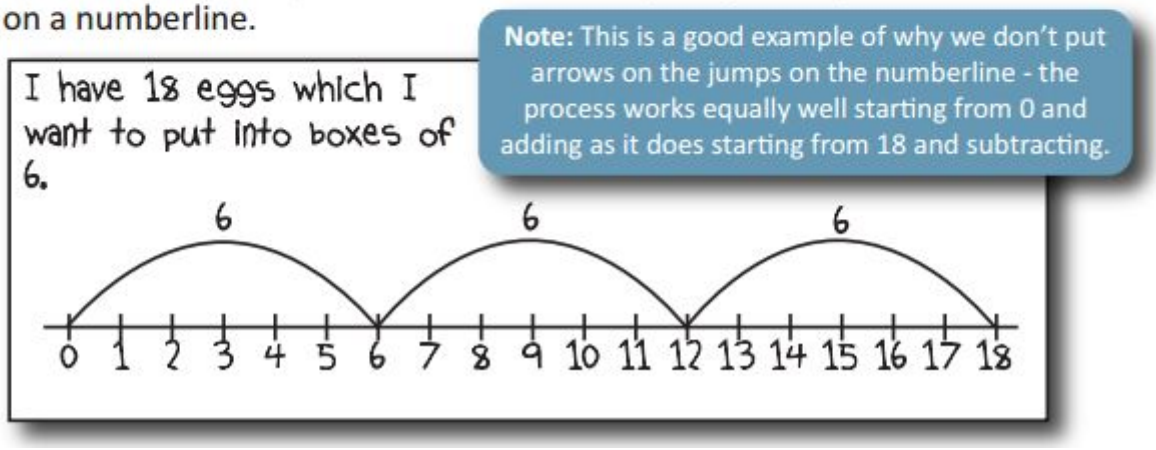
At stage 2, children develop their understanding of division as two separate processes:

- sharing (eg 12 marbles shared between 3 friends)
- grouping (eg 18 eggs are put into boxes of 6)






In the same way we can use repeated addition to show the same process, that is, we repeatedly add groups of 6 until we can't any longer. It is possible to show this on a numberline.

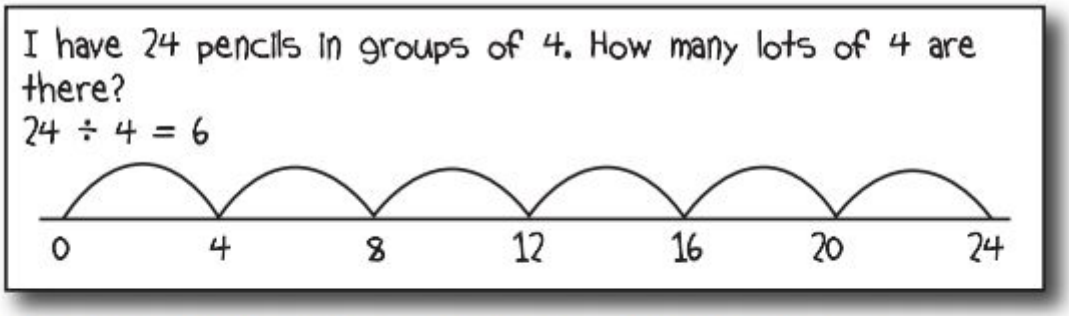



**Note:** This is a good example of why we don't put arrows on the jumps on the numberline - the process works equally well starting from 0 and adding as it does starting from 18 and subtracting.

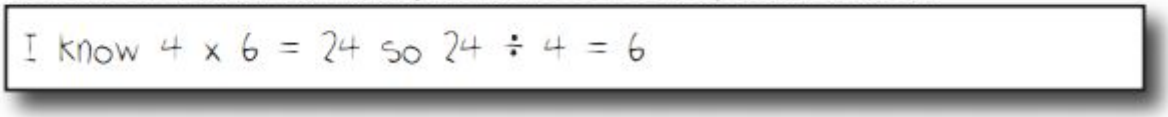
At stage 2 children experience divisions which "work". We deal with the idea of items left over, or **remainders**, at stage 3.

**Stage 3** (around Year 3)

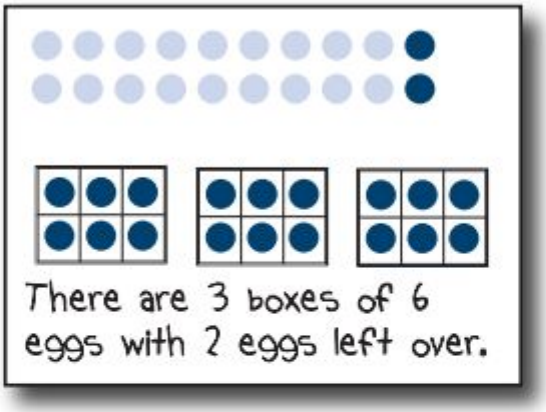
Firstly, children can carry out repeated addition on a blank numberline for a calculation with no remainder.



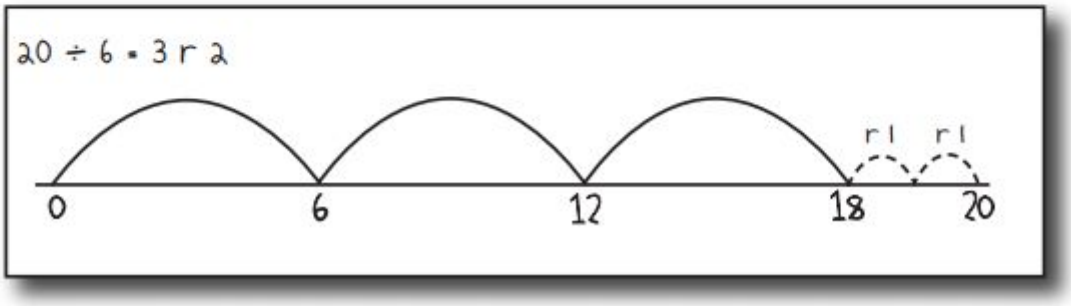

Children will use known multiplication facts to help with division.



Then they can see the effect of having a remainder. So, repeating the earlier example of putting eggs in boxes but this time with 20 eggs:



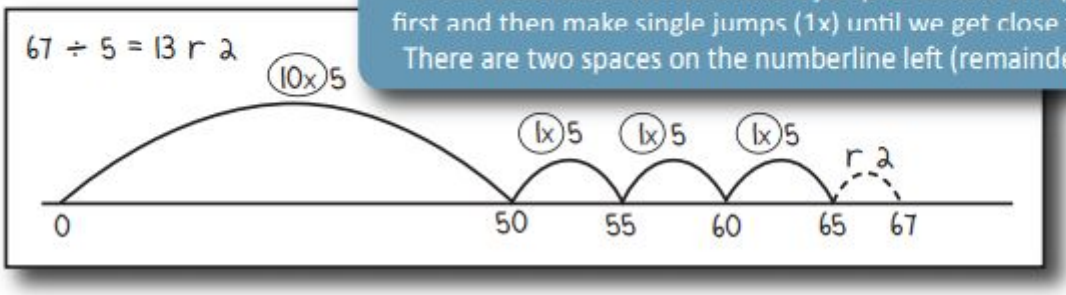

Or on a numberline:



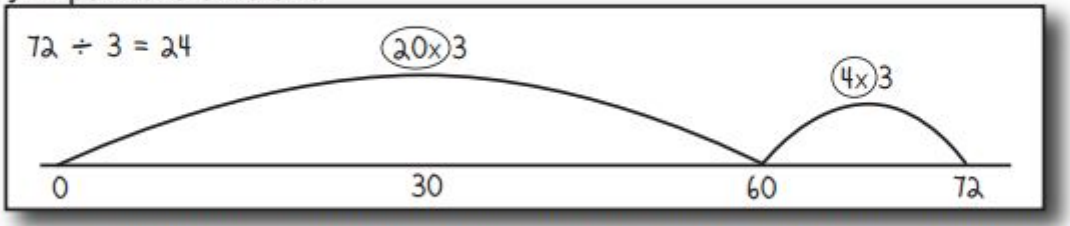

**Stage 4** (Year 4)

Stage 4 makes this process more efficient by grouping some of the individual steps into one, for example:

In this calculation we can make a jump of 10 lots of 5 (10x) first and then make single jumps (1x) until we get close to 67. There are two spaces on the numberline left (remainder 2).




By grouping more than one step as the numbers get larger we can make several larger jumps to the target number. Related multiplication facts are useful to make jumps more efficient.



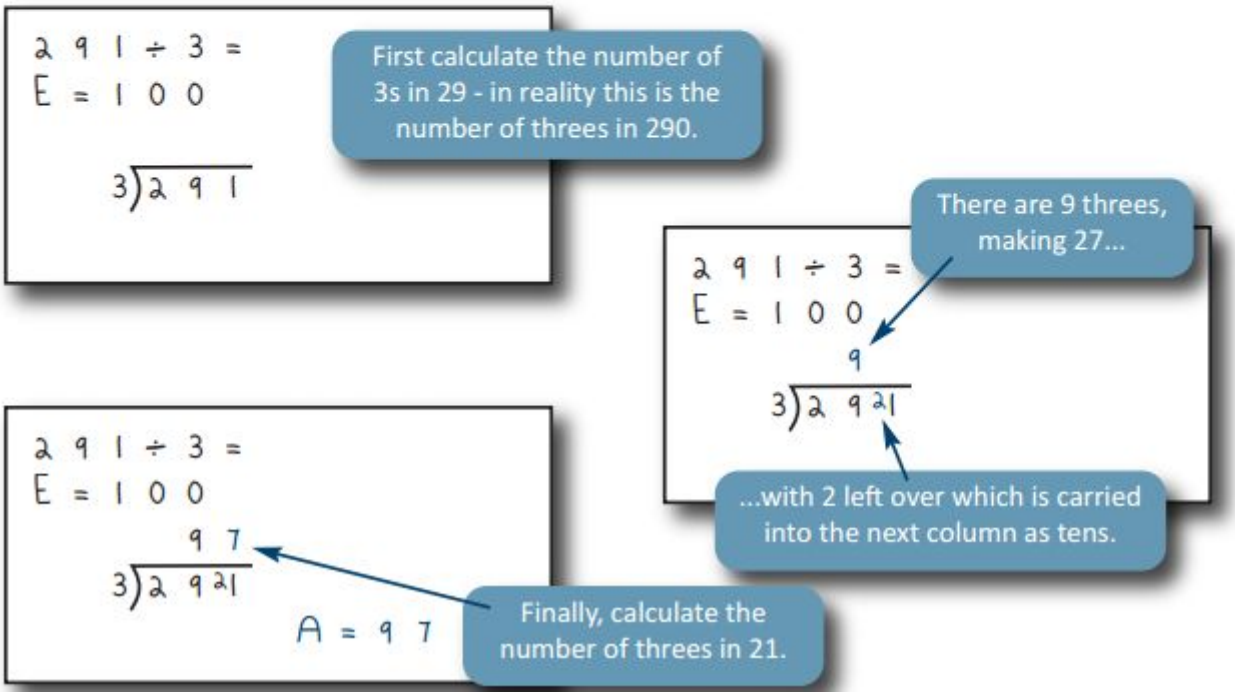

At this stage, children also divide whole numbers by 10 and 100, extending their knowledge of place value including decimals.

## Stage 5 (Year 5)

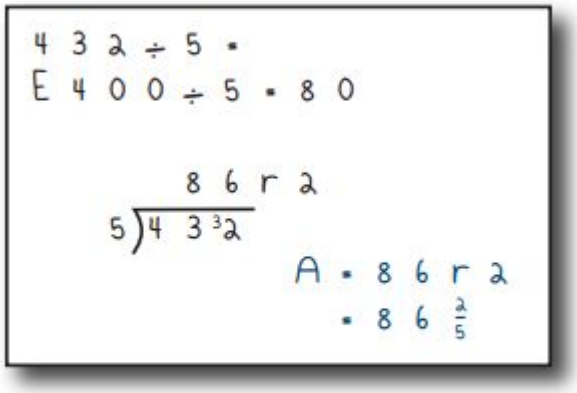
At this stage children will multiply and divide whole numbers and decimals by 10, 100 and 1000, drawing on known multiplication facts.

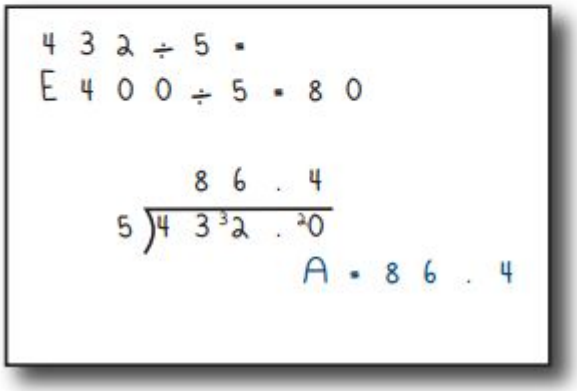
Children will continue to use division on a number line for some questions.

**Short Division:** children will begin to use formal written methods for three-digit (HTU) numbers divided by single-digit (U) numbers.




Children should be able to interpret the remainder as a fraction or decimal, for example:

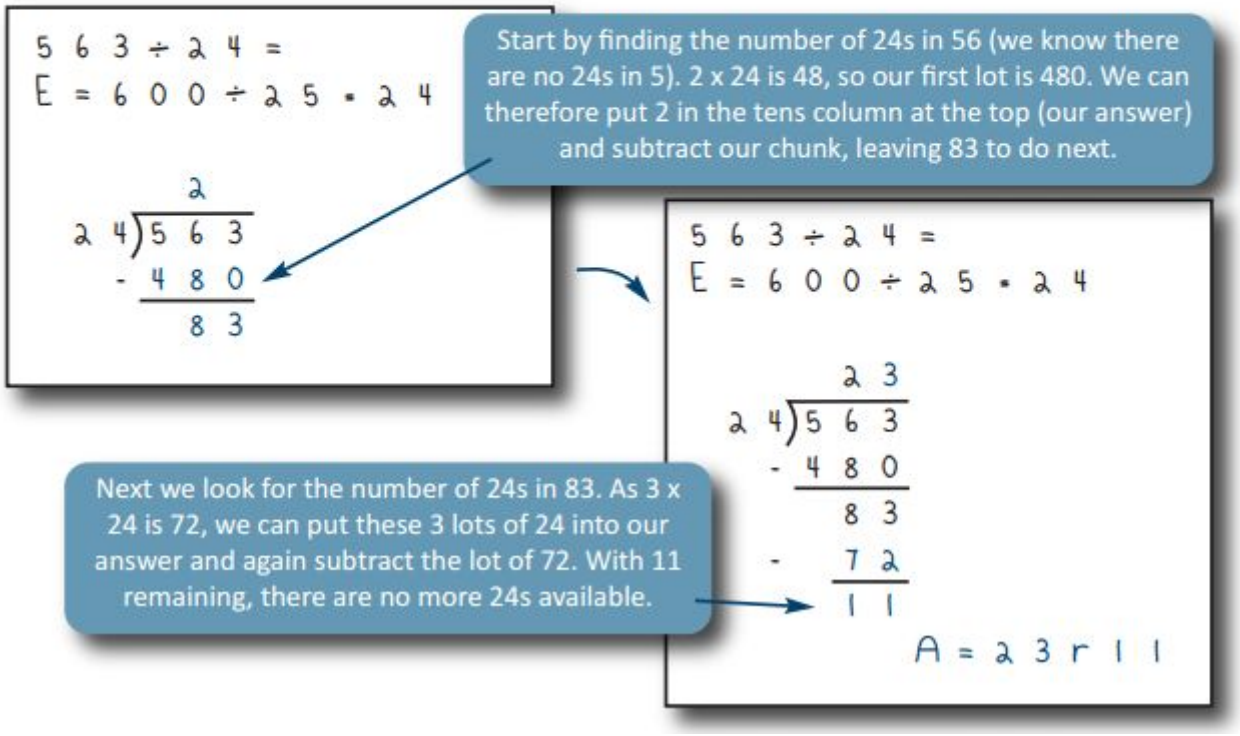


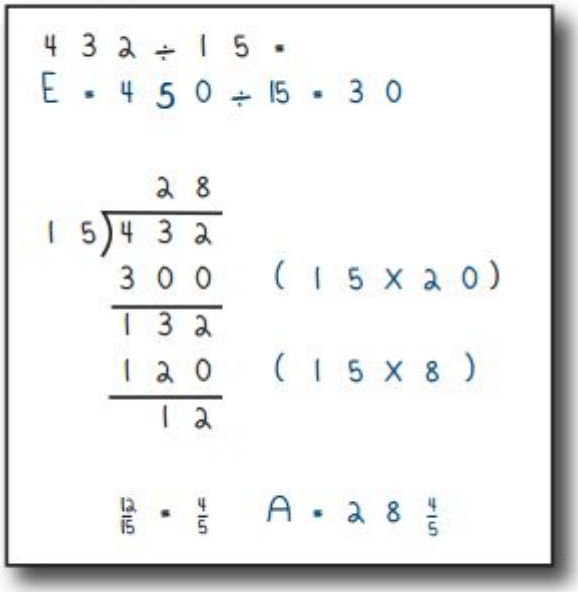
**Stage 6 (Year 6)**

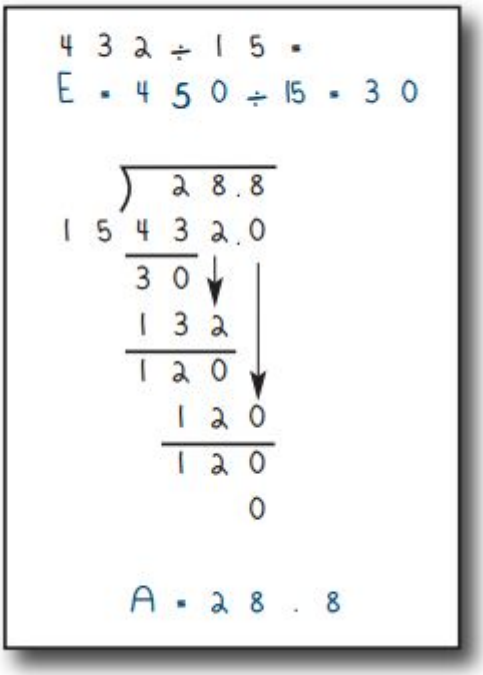
Finally, long division allows us to tackle calculations where we want to divide by a two-digit number.

Children will continue to use division on a number line for some questions.




Children should be able to interpret the remainder as a fraction or decimal, for example:

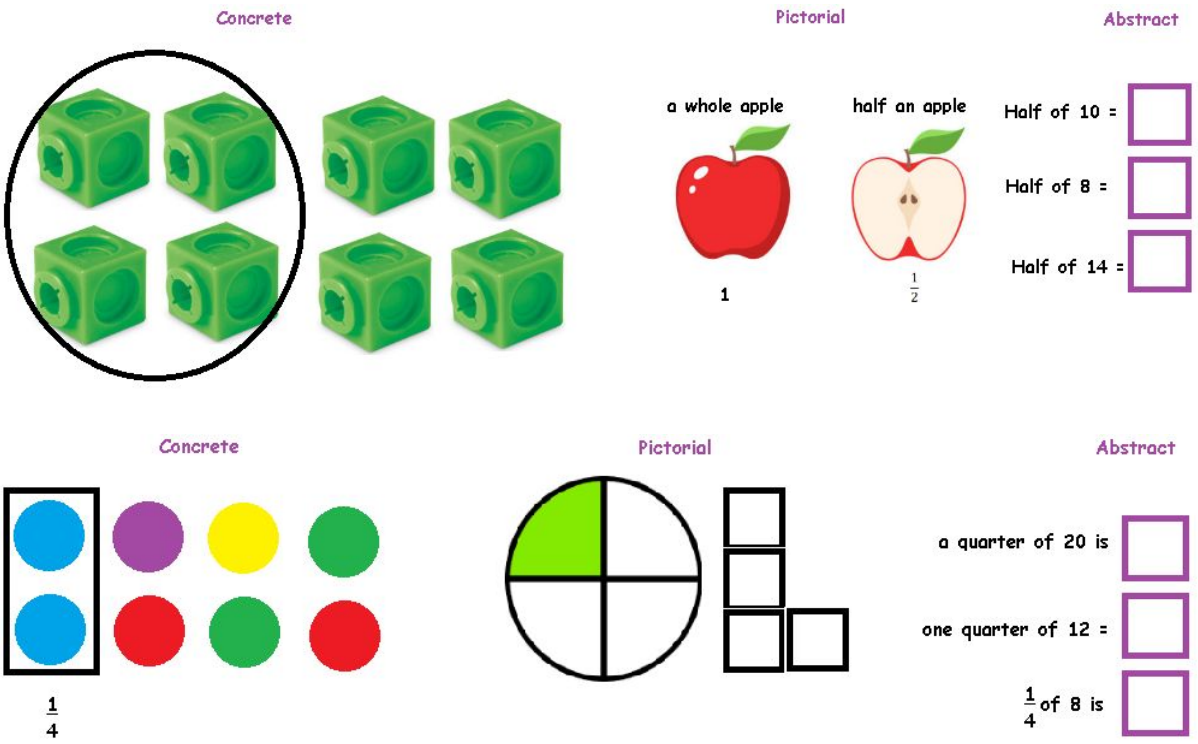


# Fractions (percentages and decimals)

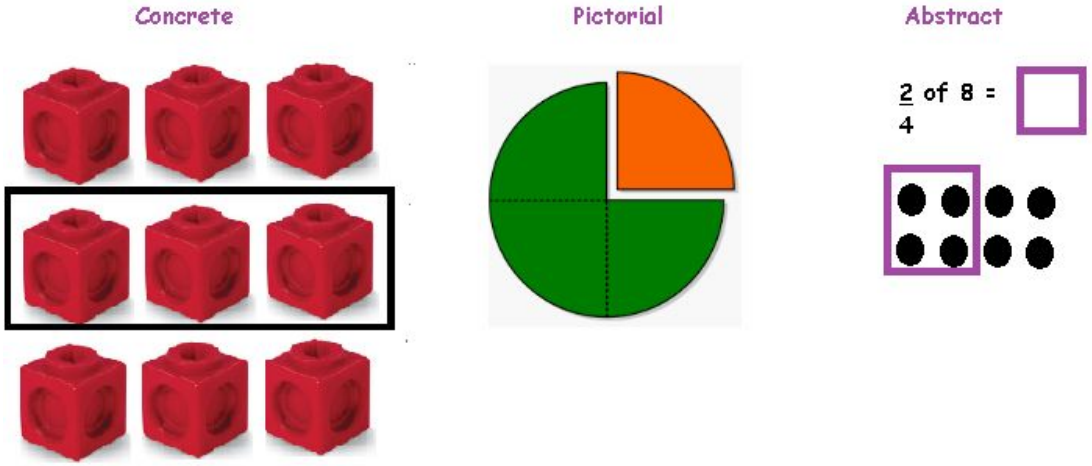
| | Pupils should be taught to |
|---|---|
| **Foundation Stage** | •recognise half |
| **Year 1** | • recognise, find and name a half as one of two equal parts of an object, shape or quantity<br>• recognise, find and name a quarter as one of four equal parts of an object, shape or quantity. |
| **Year 2** | • recognise, find, name and write fractions ½, ¼, 2/4, ¾ of a length, shape, set of objects or quantity.<br>• write simple fractions e.g. ½ of 6 = 3<br>• recognise the equivalence of 2/4 and ½ |
| **Year 3** | • count up and down in tenths; recognise that tenths arise from dividing an object into 10 equal parts and in dividing one-digit numbers or quantities by 10.<br>• recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions with small denominators.<br>• recognise and use fractions as numbers: unit fractions and non-unit fractions with small denominators.<br>• recognise and show, using diagrams, equivalent fractions with small denominators<br>• add and subtract fractions with the same denominator within one whole [for example, 5/7 + 1/7 = 6/7<br>• compare and order unit fractions, and fractions with the same denominators<br>• solve problems that involve all of the above. |
| **Year 4** | • recognise and show, using diagrams, families of common equivalent fractions<br>• count up and down in hundredths; recognise that hundredths arise when dividing an object by one hundred and dividing tenths by ten.<br>• solve problems involving increasingly harder fractions to calculate quantities, and fractions to divide quantities, including non-unit fractions where the answer is a whole number<br>• add and subtract fractions with the same denominator<br>• recognise and write decimal equivalents of any number of tenths or hundredths<br>• recognise and write decimal equivalents to ¼, ½, and ¾.<br>• find the effect of dividing a one- or two-digit number by 10 and 100, identifying the value of the digits in the answer as ones, tenths and hundredths<br>• round decimals with one decimal place to the nearest whole number<br>• compare numbers with the same number of decimal places up to two decimal places<br>• solve simple measure and money problems involving fractions and decimals to two decimal places. |
| **Year 5** | • compare and order fractions whose denominators are all multiples of the same number<br>• identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths<br>• recognise mixed numbers and improper fractions and convert from one form to the other and write mathematical statements > 1 as a mixed number [for example, ⅖ + ⅘ = 6/5 = 1 ⅕<br>• add and subtract fractions with the same denominator and denominators that are multiples of the same number.<br>• multiply proper fractions and mixed numbers by whole numbers, supported by materials and diagrams.<br>• read and write decimal numbers as fractions [for example, 0.71 = 71/100]<br>• recognise and use thousandths and relate them to tenths, hundredths and decimal equivalents<br>• round decimals with two decimal places to the nearest whole number and to one decimal place<br>• read, write, order and compare numbers with up to three decimal places<br>• solve problems involving numbers with up to three decimal places<br>• recognise the percent symbol (%) and understand that percent relates to 'number of parts per hundred', and write percentages as a fraction with denominator 100, and as a decimal<br>• solve problems which require knowing percentage and decimal equivalents of ½, ¼, ⅕, ⅖, ⅘ and those fractions with a denominator of a multiple of 10 or 25. |
| **Year 6** | • use common factors to simplify fractions; use common multiples to express fractions in the same denomination.<br>• compare and order fractions, including fractions > 1<br>• add and subtract fractions with different denominators and mixed numbers, using the concept of equivalent fractions.<br>• multiply simple pairs of fractions, writing the answer in its simplest form e.g. ¼ x ½ = 1/8<br>• divide proper fractions by whole numbers e.g. 1/3 ÷ 2 = 1/6<br>• associate a fraction with division and calculate decimal fraction equivalents [for example, 0.375] for a simple fraction e.g. 3/8<br>• identify the value of each digit in numbers given to three decimal places and multiply and divide numbers by 10, 100 and 1000 giving answers up to three decimal places<br>• multiply one-digit numbers with up to two decimal places by whole numbers<br>• use written division methods in cases where the answer has up to two decimal places<br>• solve problems which require answers to be rounded to specified degrees of accuracy<br>• recall and use equivalences between simple fractions, decimals and percentages, including in different contexts. |
| **Vocabulary** | half, quarter, equal, whole, part three-quarters, third, mixed number, whole number, fraction<br>fraction statement, fraction sentence, numerator, denominator, equivalent<br>unit fractions, non-unit fractions, simplified fractions, fraction family, mixed number<br>improper fractions, proper fractions, decimal, percent<br>simplest form, convert, ratio, |

## Stage 1 (Foundation Stage to Year 1)




## Stage 2 (Year 2)

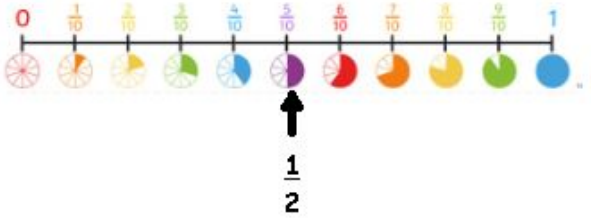
**Recognise, find, name and write the fractions ⅓, ¼, ½, ¾**
**Find ⅓, ¼, ½, ¾ of a length, shape or set of objects**



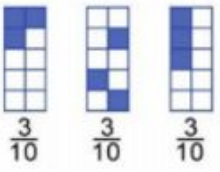

**Write simple fractions and recognise equivalence of ½**

**Count up and down in tenths: recognise that tenths arise from dividing an object into ten equal parts and in dividing one-digit numbers or quantities by 10.**

Concrete




Pictorial




Abstract

1/10 of 6 = 0.6

because

6 ÷ 10 = 0.6

**Stage 3 (Year 3)**

**Recognise, find and write fractions of a discrete set of objects: unit fractions and non-unit fractions and use fractions as numbers.**

Concrete

1/5

Pictorial

3/4

Abstract

1/5 of 15 sweets is 3

because 15 ÷ 5 = 3

2/5 of 15 sweets = 6

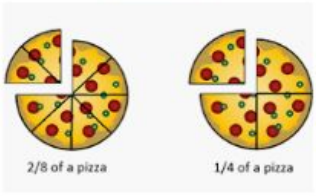
because

15 ÷ 5 = 3

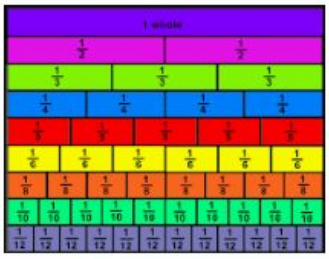
and 3 x 2 = 6

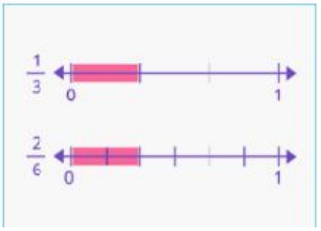
**Recognise and show using diagrams, equivalent fractions with small denominators**

Concrete

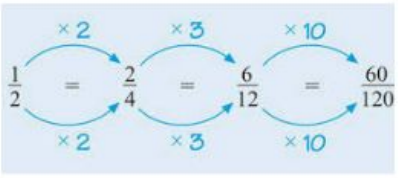



Pictorial




Abstract



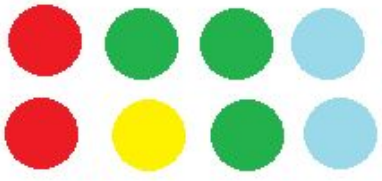

Maya says one half is the same as two quarters. Is she correct? Explain.

## Add and subtract fractions with the same denominator

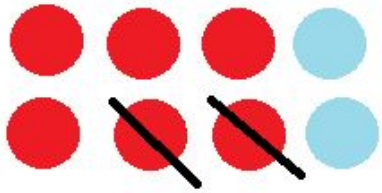
Concrete

$\frac{8}{8} - \frac{2}{8} = \frac{6}{8}$

Pictorial



$\frac{2}{6} + \frac{1}{6} + \frac{3}{6} + \frac{2}{6} = \frac{8}{8} =$ |



$\frac{6}{8} - \frac{2}{8} = \frac{4}{8}$

Abstract

$\frac{5}{7} + \frac{1}{7} = \frac{6}{7}$

$\frac{5}{8} - \frac{2}{8} = \frac{3}{8}$

## Compare and order unit fractions with the same denominators

Concrete

Pictorial

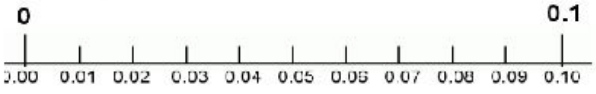
Abstract

$\frac{1}{7} < \frac{5}{7} < \frac{6}{7}$

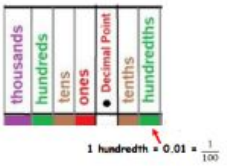
### Stage 4 (Year 4)

## Count up and down in hundredths: recognise that hundredths arise when dividing an object by 100 and dividing tenths by 10.

Concrete




Pictorial



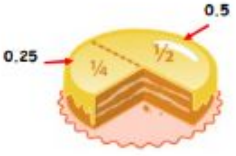

Abstract

$\frac{1}{100}$ of 60 = 0.6

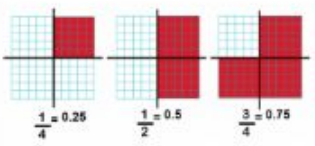
because 60 ÷ 100 = 0.6

## Recognise and write decimal equivalents to ½, ¼ and ¾ .

Concrete




Pictorial




Abstract

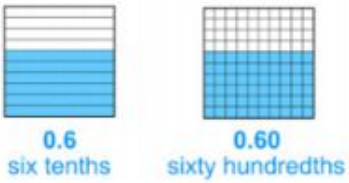
$\frac{1}{2} = 0.5$

$\frac{1}{4} = 0.25$ $\frac{3}{4} = 0.75$

## Recognise and write decimal equivalents of any number of tenths or hundredths.

**Concrete**


**Pictorial**




**Abstract**

$\frac{1}{10} = 0.1$

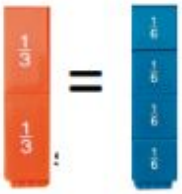
$\frac{3}{10} = 0.3$

$\frac{5}{10} = \frac{1}{2} = 0.5$

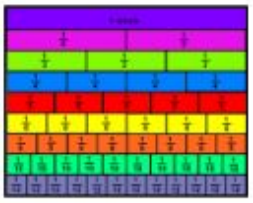
$\frac{8}{100} = 0.08$

## Recognise and show, using diagrams, families of common equivalents.

**Concrete**



**Pictorial**



**Abstract**

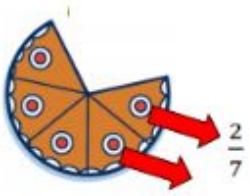
$\frac{2}{3} = \frac{4}{6}$

$\frac{3}{5} = \frac{6}{10}$

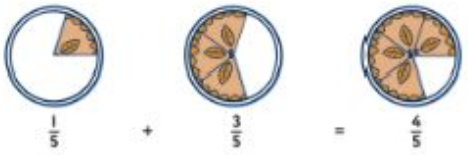
$\frac{2}{12} = \frac{1}{6}$

## Add and subtract fractions with the same denominator.

**Concrete**




**Pictorial**




**Abstract**

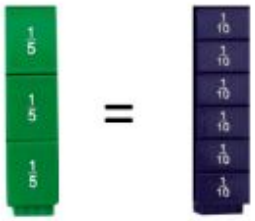
Sam eats $\frac{2}{7}$ of a whole pizza. How much does he have left?

**At this stage, answers may go over a whole.**

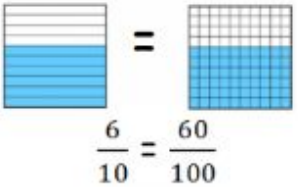
### Stage 5 (Year 5)

## Identify, name and write equivalent fractions of a given fraction, represented visually, including tenths and hundredths.

**Concrete**



**Pictorial**




**Abstract**

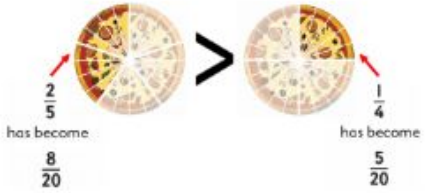
$\frac{3}{5} = \frac{6}{10} = \frac{60}{100}$

$\frac{3}{4} = \frac{75}{100}$

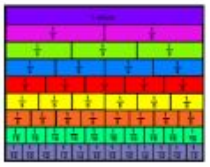
$\frac{1}{5} = \frac{2}{10} = \frac{20}{100}$

## Compare and order fractions whose denominators are all a multiple of each other.

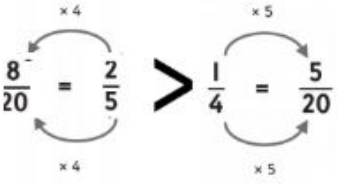
### Concrete




### Pictorial

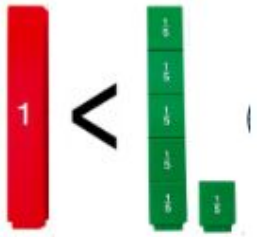


### Abstract




## Recognise mixed numbers and improper fractions. Convert from one form to the other and write mathematical statements >1 as a mixed number.

### Concrete




### Pictorial

$\frac{3}{5} + \frac{4}{5} = \frac{7}{5}$

$\frac{7}{5} = 1\frac{2}{5}$

### Abstract

$\frac{7}{2} = 3\frac{1}{2}$

because 7 ÷ 2 = 3 with one half left over

$2\frac{1}{3} = \frac{7}{3}$

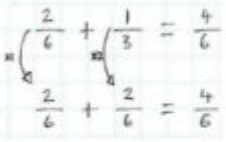
because 2 x 3 = 6 with 1 third left to add

## Add and subtract fractions where one denominator is a multiple of the other
## Adding and subtracting fractions with different denominators

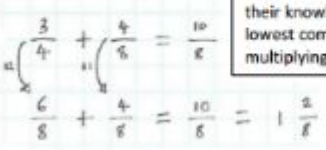
### Concrete

### Pictorial

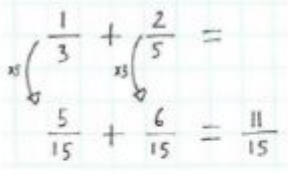
### Abstract

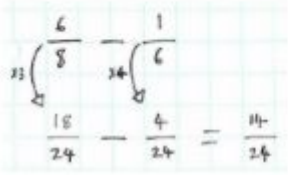



Children are converting to find the lowest common multiple for the first time – a secure understanding of equivalent fractions is therefore required. In the first instance, examples should be used that remain within a whole before practising with mixed numbers. Children draw upon their knowledge of multiples to find the lowest common multiple rather than multiplying the two denominators.




The next step, in Year 5, now requires children to convert their answer from an improper fraction to a mixed number.



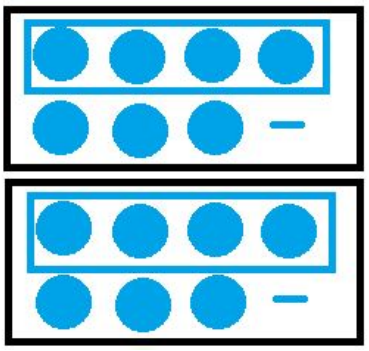




At this stage, both common denominators are converted to the lowest common multiple. Children will need to draw upon their times tables knowledge in identifying these. It may be useful to provide additional support through the inclusion of an arrow indicating what the numerator and denominator are being multiplied by when finding the equivalent fraction.

## Multiply proper fractions and mixed numbers by whole numbers.

Concrete

Pictorial



Abstract

$1\frac{4}{5} \times 4 = 4\frac{16}{5} = 7\frac{1}{5}$

because

$1 \times 4 = 4$

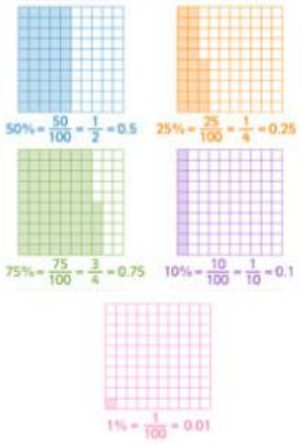
$\frac{4}{5} \times 4 = \frac{16}{5}$

$\frac{16}{5} = 3\frac{1}{5}$ as $5 \times 3 = 15$ and $15+1 = 16$

## Recognise the % symbol and understand the meaning: write % as a fraction, percentage and decimal.

Concrete


Pictorial



Abstract

$\frac{1}{10} = 0.1 = 10\%$

$\frac{1}{100} = 0.01 = 1\%$

$\frac{25}{100}$ $\frac{1}{4}$ $= 0.25 = 25\%$

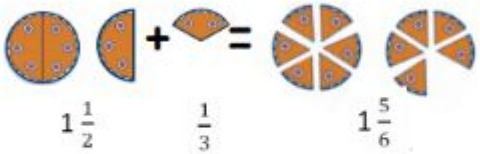
To find 10 %, divide by 10

To find 1%, divide by 100

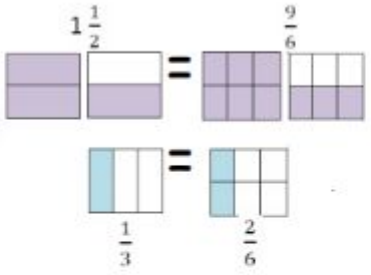
### Stage 6 (Year 6)

## Add and subtract fractions with different denominators and mixed numbers using the concept of equivalent fractions.

Concrete




Pictorial




Abstract

$1\frac{1}{2} + \frac{1}{3} = 1\frac{5}{6}$

because $1\frac{1}{2} = \frac{3}{2}$

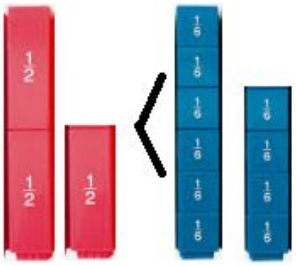
$\frac{3}{2} = \frac{9}{6}$ and $\frac{1}{3} = \frac{2}{6}$

so $\frac{9}{6} + \frac{2}{6} = \frac{11}{6} = 1\frac{5}{6}$

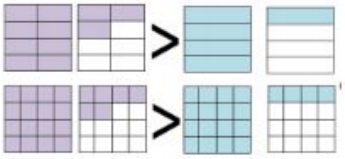

The final stage requires children to again identify the lowest common multiple. A possible misconception here is that children may, in finding the equivalent fraction, multiply the whole number.

## Compare and order fractions (including fractions greater than one)

Concrete



Pictorial



Abstract

$$\frac{1}{6} \xrightarrow{\times 5} \frac{5}{30} \qquad \frac{2}{3} \xrightarrow{\times 10} \frac{20}{30} \qquad \frac{3}{5} \xrightarrow{\times 6} \frac{18}{30}$$

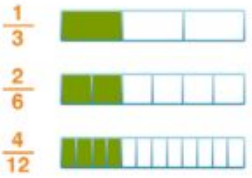
So from smallest to biggest they would be ...

$$\frac{1}{6} \quad \frac{3}{5} \quad \frac{2}{3}$$

## Use common factors to simplify fractions; use common multiples to express fractions in the same denomination.

Concrete


Pictorial



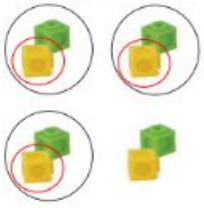

Abstract

$$\frac{18}{36} \overset{\div 3}{=} \frac{6}{12} \overset{\div 6}{=} \frac{1}{2}$$

## Multiply simple pairs of proper fractions, writing the answer in its simplest form.

Concrete

$\frac{1}{2}$ of $\frac{3}{4}$



Pictorial


$\frac{1}{2}$ of $\frac{6}{8}$

Abstract

$$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$$

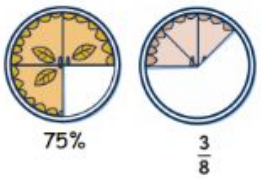
$$\frac{3}{4} \times \frac{2}{6} = \frac{3 \times 2}{4 \times 6} = \frac{6}{24} = \frac{1}{4}$$

## Recall and use equivalences between simple fractions, decimals and percentages.

### Concrete


### Pictorial

Would you prefer 75% or $\frac{3}{8}$ of a pie?





### Abstract

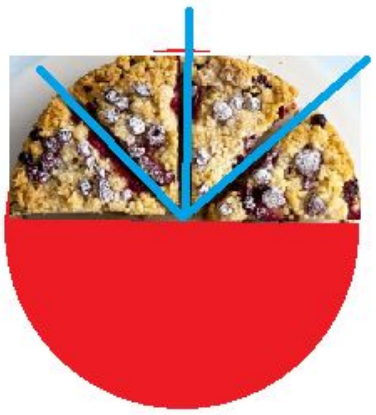
Hans scored forty out of eighty on his spelling test. Mia scored 40%. Who scored more? Why

$\frac{40}{80} = \frac{1}{2} = 50\%$

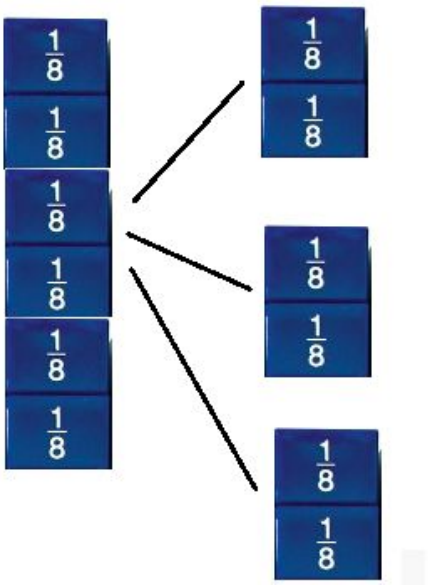
so Hans scored more.

## Divide proper proper fractions by whole numbers.

### Concrete



### Pictorial




### Abstract

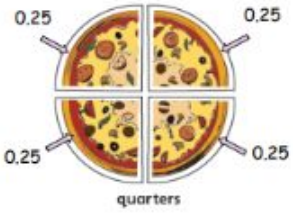
$\frac{1}{2} \div 3 = \frac{1}{8}$

as

$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$

## Associate fractions with division and calculate decimal fraction equivalents

### Concrete




### Pictorial

3 slices of pie out of 8

### Abstract

3 'out of' 8 is the same as 3 divided by 8.

$3 \div 8 = 0.375$

so $\frac{3}{8} = 0.375$

# Glossary abc

**array** - an organised collection of objects, counters or symbols, for example arranged in rows and columns 

**bridging** - the process of using a multiple of 10 or 100 as part of an addition or subtraction calculation, for example 45 + 13 can be thought of as 45 + 5 (50) + 8

**decomposition** - the standard written method for subtraction (see p10)

**difference** - the amount by which one number is greater than another - i.e. the result of a subtraction; the difference between 5 and 9 is 4

**grid method** - a method of calculating multiplication by separating the calculation into sections, each of which easier than the whole (see p12)

**least significant digits** - the digits with least value - usually the units

**number track** - a line of numbers used for counting or calculating, each section represents one number

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|

**numberline** - a line where numbers are represented by points on it; numberlines always run from left to right


**partitioning** - separating a number into its different parts, eg 25 can be partitioned into 20 and 5

**remainder** - the amount left over in a division which cannot be grouped or shared equally