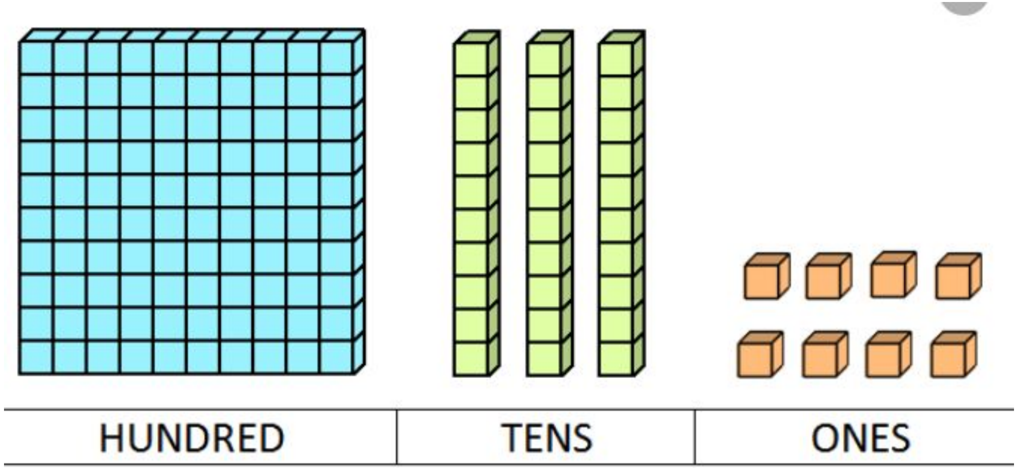
Number

Place value, addition and subtraction, multiplication and division, fractions, decimals, percentages, ratio and algebra

Place value and counting



Sorting

set (KS1): A well-defined collection of objects (called members or elements).

sort (KS1): To classify a set of entities into specified categories.

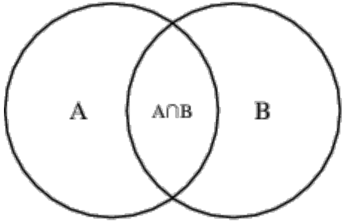
Carroll diagram (KS1): A sorting diagram named after Lewis Carroll, author and mathematician, in which numbers (or objects) are classified as having a certain property or not having that property

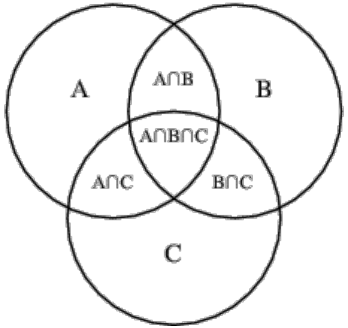
venn diagram (KS2): A simple visual diagram to describe used to describe the relationships between two sets. With two or three sets each set is often represented by a circular region. The intersection of two sets is represented by the overlap region between the two sets. With more than three sets Venn diagrams can become very complicated. The boundary of the Venn Diagram represents the Universal Set of interest.

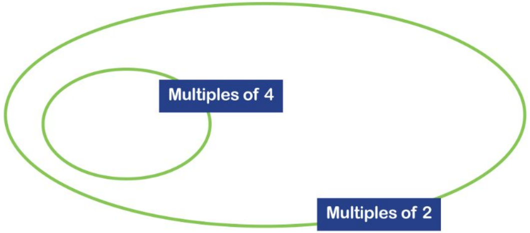
compare (KS1/2/3): In mathematics when two entities (objects, shapes, curves, equations etc.) are compared one is looking for points of similarity and points of difference as far as mathematical properties are concerned.

8 22 18 49 100

	Even numbers	Odd numbers
Numbers in the 7x table		
Numbers not in the 7x table		







Numerals

Roman numerals (KS2): The Romans used the following capital letters to denote cardinal numbers: I for 1; V for 5; X for 10; L for 50; C for 100; D for 500; M for 1000.
Multiples of one thousand are indicated by a bar over a letter, so for example V with a bar over it means 5000. Other numbers are constructed by forming the shortest sequence with this total, with the proviso that when a higher denomination follows a lower denomination the latter is subtracted from the former.

Examples: III =3; IV = 4; XVII =17; XC = 90; CX =110; CD = 400; MCMLXXII = 1972. A particular feature of the Roman numeral system is its lack of a symbol for zero and, consequently, no place value structure. As such it is very cumbersome to perform calculations in this number system.

numeral (KS1): A symbol used to denote a number. The Roman numerals I, V, X, L, C, D and M represent the numbers one, five, ten, fifty, one hundred, five hundred and one thousand. The Arabic numerals 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9 are used in the Hindu-Arabic system giving numbers in the form that is widely used today.

Roman Numerals: 1 - 1000

I	V	X	L	C	D	M
1	5	10	50	100	500	1000

1	I
2	II
3	III
4	IV
5	V
6	VI
7	VII
8	VIII
9	IX
10	X

11	X1
20	XX
30	XXX
40	XL
50	L
60	LX
70	LXX
80	LXXX
90	XC
100	C

200	CC
300	CCC
400	CD
500	D
600	DC
700	DCC
800	DCCC
900	CM
1000	M
1001	MI

Counting

consecutive (KS1): Following in order. Consecutive numbers are adjacent in a count. Examples: 5, 6, 7 are consecutive numbers. 25, 30, 35 are consecutive multiples of 5 multiples of 5. In a polygon, consecutive sides share a common vertex and consecutive angles share a common side.

count (verb) (KS1): The act of assigning one number name to each of a set of objects (or sounds or movements) in order to determine how many objects there are. In order to count reliably children need to be able to: • Understand that the number words come in a fixed order • Say the numbers in the correct sequence; • Organise their counting (e.g. say one number for each object and keep track of which things they have counted); • Understand that the final word in the count gives the total • Understand that the last number of the count remains unchanged irrespective of the order (conservation of number)

concrete objects (KS1): Objects that can be handled and manipulated to support understanding of the structure of a mathematical concept. Materials such as Dienes (Base 10 materials), Cuisenaire, Numicon, pattern blocks are all examples of concrete objects

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Place value

digit (KS1): One of the symbols of a number system most commonly the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. Examples: the number 29 is a 2-digit number; there are three digits in 2.95. The position or place of a digit in a number conveys its value.

integer (KS2): Any of the positive or negative whole numbers and zero. Example: …- 2, -1, 0, +1, +2 … The integers form an infinite set; there is no greatest or least integer.

placeholder (KS2): In decimal notation, the zero numeral is used as a placeholder to denote the absence of a particular power of 10. Example: The number 105.07 is a shorthand for 1 × 102 + 0 × 101 + 5 × 100 + 0 × 10−1 + 7 × 10−2 .

place value (KS1): The value of a digit that relates to its position or place in a number. Example: in 1482 the digits represent 1 thousand, 4 hundreds, 8 tens and 2 ones respectively; in 12.34 the digits represent 1 ten, 2 ones, 3 tenths and 4 hundredths respectively

positive number (KS2): A number greater than zero. Where a point on a line is labelled 0 positive numbers are all those to the left of the zero and are read 'positive one, positive two, positive three' etc. See also directed number and negative number.

partition (KS1): To separate a set into subsets. 2. To split a number into component parts. Example: the two-digit number 38 can be partitioned into 30 + 8 or 19 + 19. 3. A model of division. Example: 21 ÷ 7 is treated as ‘how many sevens in 21?’
negative integer (KS2) An integer less than 0. Examples: -1, -2, -3 etc.

negative number (KS2): A number less than zero. Example: − 0.25. Where a point on a line is labelled 0 negative numbers are all those to the left of the zero on a horizontal number line. Commonly read aloud as ‘minus or negative one, minus or negative two’ etc. the use of the word ‘negative’ often used in preference to ‘minus’ to distinguish the numbers from operations upon them.

Number

rational number (KS2): A number that is an integer or that can be expressed as a fraction whose numerator and denominator are integers, and whose denominator is not zero. Examples: − 1, ⅓, 3 /5, 9, 235. Rational numbers, when expressed as decimals, are recurring decimals or finite (terminating) decimals. Numbers that are not rational are irrational. Irrational numbers include √5 and π which produce infinite, non-recurring decimals. R

real numbers: A number that is rational or irrational. Real numbers are those generally used in everyday contexts, but in mathematics, or the physical sciences, or in engineering, or in electronics the number system is extended to include what are known as complex numbers. In school mathematics to key stage 4 all the mathematics deals with real numbers. Integers form a subset of the real numbers.

natural number (KS2): The counting numbers 1, 2, 3, ... etc. The positive integers. The set of natural numbers is usually denoted by N.

ordinal number (KS1): A term that describes a position within an ordered set. Example: first, second, third, fourth ... twentieth etc.

cardinal number (KS1): A cardinal number denotes quantity, as opposed to an ordinal number which denotes position within a series. 1, 2, 5, 23 are examples of cardinal numbers First (1st), second (2nd), third (3rd) etc denote position in a series, and are ordinals.

Number

number bond (KS1): A pair of numbers with a particular total e.g. number bonds for ten are all pairs of whole numbers with the total 10.

number line (KS1): A line where numbers are represented by points upon it.

odd number (KS2): An integer that has a remainder of 1 when divided by 2.

even number (KS1): An integer that is divisible by 2.

number sentence (KS1): A mathematical sentence involving numbers. Examples: $3 + 6 = 9$ and $9 > 3$

odd number (KS2): An integer that has a remainder of 1 when divided by 2.

zero (KS1): Nought or nothing; zero is the only number that is neither positive nor negative. 2. Zero is needed to complete the number system. In our system of numbers : $a – a = 0$ for any number a. $a + (–a) = 0$ for any number a; $a + 0 = 0 + a = a$ for any number a; $a – 0 = a$ for any number a; $a × 0 = 0 × a = 0$ for any number a; division by zero is not defined as it leads to inconsistency. 3. In a place value system, a place-holder. Example: 105. 4. The cardinal number of an empty set.

infinite (KS1): Of a number, always bigger than any (finite) number that can be thought of. Of a sequence or set, going on forever. The set of integers is an infinite set.

round (verb) (KS2): In the context of a number, express to a required degree of accuracy. Example: 543 rounded to the nearest 10 is 540. row A horizontal arrangement.

Number

sequence (KS1): A succession of terms formed according to a rule. There is a definite relation between one term and the next and between each term and its position in the sequence. Example: 1, 4, 9, 16, 25 etc.

triangular number (KS1): 1. A number that can be represented by a triangular array of dots with the number of dots in each row from the base decreasing by one. Example: The triangular number 10 represented as a triangular array of dots. 2. A number in the sequence 1, 1 + 2, 1 + 2 + 3, 1 + 2 + 3 + 4 etc. 55 is a triangular number since it can be expressed as, 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10.

maximum value (in a noncalculus sense) (KS1): The greatest value.

estimate (KS2): Verb: To arrive at a rough or approximate answer by calculating with suitable approximations for terms or, in measurement, by using previous experience. Noun: A rough or approximate answer.

minimum value (in a noncalculus sense) (KS1): The least value.

inequality (KS1): When one number, or quantity, is not equal to another. Statements such as a ≠ b, a b or a≥b are inequalities. The inequality signs in use are: ≠ means ‘not equal to’; A ≠ B means ‘A is not equal to B" < means ‘less than’; A < B means ‘A is less than B’ > means ‘greater than’; A > B means ‘A is greater than B’ ≤ means ‘less than or equal to’; A ≤ B means ‘A is less than or equal to B’ ≥ means ‘greater than or equal to’; A ≥ B means ‘A is greater than or equal to B’

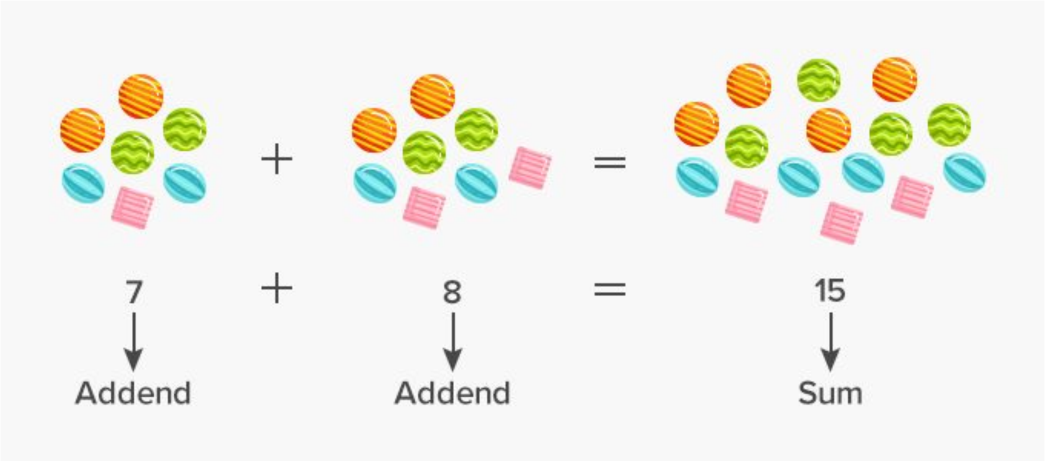
addition and subtraction

Addition

Addend (KS1): A number to be added to another. See also dividend, subtrahend and multiplicand.

sum (KS1): The result of one or more additions.



complement (in addition) (KS2): In addition, a number and its complement have a given total. Example: When considering complements in 100, 67 has the complement 33, since 67 + 33 = 100

total (KS1): The aggregate. Example: the total population - all in the population. 2. The sum found by adding.

plus (KS1): A name for the symbol +, representing the operation of addition.

Addition

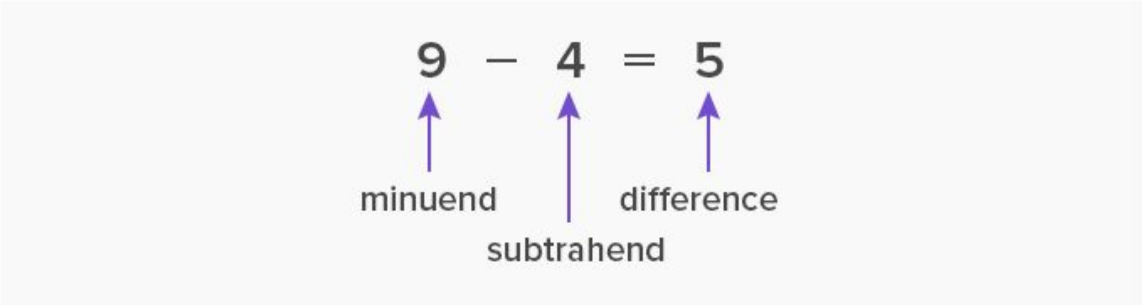
addition (KS1): The binary operation of addition on the set of all real numbers that adds one number of the set to another in the set to form a third number which is also in the set. The result of the addition is called the sum or total. The operation is denoted by the + sign. When we write 5 + 3 we mean 'add 3 to 5'; we can also read this as '5 plus 3'. In practice the order of addition does not matter: The answer to 5 + 3 is the same as 3 + 5 and in both cases the sum is 8. This holds for all pairs of numbers and therefore the operation of addition is said to be commutative. To add three numbers together, first two of the numbers must be added and then the third is added to this intermediate sum. For example, (5 + 3) + 4 means 'add 3 to 5 and then add 4 to the result' to give an overall total of 12. Note that 5 + (3 + 4) means 'add the result of adding 4 to 3 to 5' and that the total is again 12. The brackets indicate a priority of sub-calculation, and it is always true that (a + b) + c gives the same result as a + (b + c) for any three numbers a, b and c. This is the associative property of addition. Addition is the inverse operation to subtraction, and vice versa. There are two models for addition: Augmentation is when one quantity or measure is increased by another quantity. i.e. "I had £3.50 and I was given £1, then I had £4.50". Aggregation is the combining of two quantities or measures to find the total. E.g. "I had £3.50 and my friend had £1, we had £4.50 altogether.

Subtraction

difference (KS1): In mathematics (as distinct from its everyday meaning), difference means the numerical difference between two numbers or sets of objects and is found by comparing the quantity of one set of objects with another. e.g. the difference between 12 and 5 is 7; 12 is 5 more than 7 or 7 is 5 fewer than 12. Difference is one way of thinking about subtraction and can, in some circumstances, be a more helpful image for subtraction than 'takeaway' – e.g. 102 - 98

subtrahend (KS1): A number to be subtracted from another. See also Addend, dividend and multiplicand.

Minuend (KS1): The first number in a subtraction. The number from which another number (the Subtrahend) is to be subtracted.



subtract (KS1): Carry out the process of subtraction

take away (KS1): Subtraction as reduction 2. Remove a number of items from a set.

minus (KS1) A name for the symbol −, representing the operation of subtraction.

Subtraction

decomposition (KS2): See subtraction by decomposition. deductive reasoning (KS2) Deduction is typical mathematical reasoning where the conclusion follows necessarily from a set of premises (as far as the curriculum goes these are the rules of arithmetic and their generalisation in algebra, and the rules relating to lines, angles, triangles, circles etc. in geometry); if the premises are true then following deductive rules the conclusion must also be true.

subtraction by decomposition (KS2): A method of calculation used in subtraction and particularly linked with one of the main columnar methods for subtraction. In this method the number to be subtracted from (the minuend) is re-partitioned, if necessary, in order that each digit of the number to be subtracted (the subtrahend) is smaller than its corresponding digit in the minuend. e.g. in 739 – 297, only the digits in the hundreds and the ones columns are bigger in the minuend than the subtrahend. By re-partitioning 739 into 6 hundreds, 13 tens and 9 ones each separate subtraction can be performed simply, i.e.: 9 – 7 13 (tens) – 9 (tens) and 6 (hundreds) – 2 (hundreds). subtraction by equal addition A method of calculation used in subtraction and particularly linked with one of the main columnar methods for subtraction. This method relies on the understanding that adding the same quantity to both the minuend and the subtrahend retains the same difference. This is a useful technique when a digit in the subtrahend is larger than its corresponding digit in the minuend. E.g. in the example below, 7 > 2, therefore 10 has been added to the 2 (in the ones place) of the minuend to make 12 (ones) and also added to the 5 (tens) of the subtrahend to make 60 (or 6 tens) before the first step of the calculation can be completed. Similarly 100 has been added to the 3 (tens) of the minuend to make 13 (tens) and also added to the 4 (hundreds) of the subtrahend to make 5 (hundred). 932-457 becomes Answer: 475

Multiplication

Multiplication

multiplication (KS1) Multiplication (often denoted by the symbol "×") is the mathematical operation of scaling one number by another. It is one of the four binary operations in arithmetic. Because the result of scaling by whole numbers can be thought of as consisting of some number of copies of the original, whole-number products greater than 1 can be computed by repeated addition; for example, 3 multiplied by 4 (often said as "3 times 4") can be calculated by adding 4 copies of 3 together: 3 x 4 = 3 + 3 + 3 + 3 = 12 Here 3 and 4 are the "factors" and 12 is the "product". Multiplication is the inverse operation of division, and it follows that 7 ÷ 5 × 5 = 7 Multiplication is commutative, associative and distributive over addition or subtraction.

multiplicand (KS1) A number to be multiplied by another. e.g. in 5 × 3, 5 is the multiplicand as it is the number to be multiplied by 3.

Multiplier (KS1) A quantity by which a given number (the multiplicand) is to be multiplied

multiply (KS1) Carry out the process of multiplication.

product (KS1): The result of multiplying one number by another. Example: The product of 2 and 3 is 6 since 2 × 3 = 6.

multiplication table (KS1) An array setting out sets of numbers that multiply together to form the entries in the array.

multiplicative reasoning (KS2) Multiplicative thinking is indicated by a capacity to work flexibly with the concepts, strategies and representations of multiplication (and division) as they occur in a wide range of contexts. For example, from this: 3 bags of sweets, 8 sweets in each bag. How many sweets? To this and beyond: Julie bought a dress in a sale for £49.95 after it was reduced by 30%. How much would she have paid before the sale?

Multiplication

array (KS1): An ordered collection of counters, numbers etc. in rows and columns.

double (KS1): To multiply by 2. Example: Double 13 is (13 × 2) = 26. 2. The number or quantity that is twice another. Example: 26 is double 13. In this context, a 'near double' is one away from a double. Example: 27 is a near double of 13 and of 14. (N.B. spotting near doubles can be a useful mental calculation strategy e.g. seeing 25 + 27 as 2 more than double 25.

reciprocal (KS2): The multiplicative inverse of any non-zero number. Any non-zero number multiplied by its reciprocal is equal to 1. In symbols $x \times 1/x = 1$, for all $x \neq 0$. Multiplying by $1/x$ is the same as dividing by x, and since division by zero is not defined zero has to be excluded from all other numbers that all have a reciprocal.

scale (verb) (KS2): To enlarge or reduce a number, quantity or measurement by a given amount (called a scale factor). e.g. to have 3 times the number of people in a room than before; to find a quarter of a length of ribbon; to find 75% of a sum of money.

repeated addition (KS1): The process of repeatedly adding the same number or amount. One model for multiplication. Example 5 + 5 + 5 + 5 = 5 x 4.

inverse operations (KS1): Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. 5 + 6 – 6 = 5. Multiplication and division are inverse operations e.g. 6 × 10 ÷ 10 = 6.

Multiplication

index notation (KS2): The notation in which a product such as a × a × a s recorded as a^3. In this example the number 3 is called the index (plural indices) and the number represented by a is called the base.

correspondence problems (KS2): Correspondence problems are those in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

power (of ten) (KS2): 100 (i.e. 10^2 or 10 x 10) is the second power of 10, 1000 (i.e.10^3 or 10 x 10 x 10) is the third power of 10 etc. Powers of other numbers are defined in the same way.

reciprocal (KS2): The multiplicative inverse of any non-zero number. Any non-zero number multiplied by its reciprocal is equal to 1. In symbols x × 1/x = 1, for all x ≠ 0. Multiplying by 1/x is the same as dividing by x, and since division by zero is not defined zero has to be excluded from all other numbers that all have a reciprocal.

order of magnitude (KS2): The approximate size, often given as a power of 10. Example of an order of magnitude calculation: 95 × 1603 ÷ 49 ≈ 102 × 16 × 102 ÷(5 × 101) ≈ 3 × 103

Properties of multiplication

Commutative Property

$5 \times 3 = 3 \times 5$

Associative Property

$(5 \times 3) \times 2 = 5 \times (3 \times 2)$

Distributive Property

$5 \times (4 + 3) = (5 \times 4) + (5 \times 3)$

Compensation Property

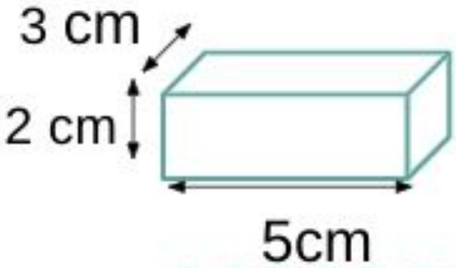
$12 \times 9 =$

Think of as $12 \times 10 = 120$

Now adjust:

Added on 1×12 so now need to subtract

$120 - 12 = 10$



Multiplication: properties of number (multiples, factors, primes, square and cube numbers …)

multiple (KS1): For any integers a and b, a is a multiple of b if a third integer c exists so that a = bc Example: 14, 49 and 70 are all multiples of 7 because 14 = 7 x 2, 49 = 7 x 7 and 70 = 7 x 10.. -21 is also a multiple of 7 since -21 = 7 ×-3.

common multiple (KS2): An integer which is a multiple of a given set of integers, e.g. 24 is a common multiple of 2, 3, 4, 6, 8 and 12.

least common multiple (LCM) (KS3): The common multiple of two or more numbers, which has the least value. Example: 3 has multiples 3, 6, 9, 12, 15, 18, 21, 24 …, 4 has multiples 4, 8, 12, 16, 20, 24 … and 6 has multiples 6, 12, 18, 24, 30 …. The common multiples of 3, 4 and 6 include 12, 24 and 36. The least common multiple of 3, 4 and 6 is 12.

factor (KS2): When a number, or polynomial in algebra, can be expressed as the product of two numbers or polynomials, these are factors of the first. Examples: 1, 2, 3, 4, 6 and 12 are all factors of 12 because 12 = 1 × 12 = 2 × 6 = 3 × 4: (x – 1) and (x + 4) are factors of (x2 + 3x − 4) because (x – 1)(x + 4) = (x2 + 3x – 4)

common factor (KS2): A number which is a factor of two or more other numbers, for example 3 is a common factor of the numbers 9 and 30 This can be generalised for algebraic expressions: for example (x – 1) is a common factor of $(x – 1)^2$ and (x – 1)(x + 3).

highest common factor (HCF) (KS3): The common factor of two or more numbers which has the highest value. Example: 16 has factors 1, 2, 4, 8, 16. 24 has factors 1, 2, 3, 4, 6, 8, 12, 24. 56 has factors 1, 2, 4, 7, 8, 14, 28, 56. The common factors of 16, 24 and 56 are 1, 2, 4 and 8. Their highest common factor is 8.

Multiplication: properties of number (multiples, factors, primes, square and cube numbers ...)

prime number (KS2): A whole number greater than 1 that has exactly two factors, itself and 1. Examples: 2 (factors 2, 1), 3 (factors 3, 1). 51 is not prime (factors 51, 17, 3, 1).

prime factor (KS2): The factors of a number that are prime. Example: 2 and 3 are the prime factors of 12 ($12 = 2 \times 2 \times 3$). See also factor. prime factor

decomposition (KS2): The process of expressing a number as the product of factors that are prime numbers. Example: $24 = 2 \times 2 \times 2 \times 3$ or $2^3 \times 3$. Every positive integer has a unique set of prime factors.

factorise (KS2): To express a number or a polynomial as the product of its factors. Examples:
Factorising 12: $12 = 1 \times 12 = 2 \times 6 = 3 \times 4$
The factors of 12 are 1, 2, 3, 4, 6 and 12.
12 may be expressed as a product of its prime factors: $12 = 2 \times 2 \times 3$
Factorising $x^2 - 4x - 21$: $x^2 - 4x - 21 = (x + 3)(x - 7)$
The factors of $x^2 - 4x - 21$ are $(x + 3)$ and $(x - 7)$

Multiplication: properties of number (multiples, factors, primes, square and cube numbers …)

square number (KS2): A number that can be expressed as the product of two equal numbers. Example 36 = 6 x 6 and so 36 is a square number or “6 squared”. A square number can be represented by dots in a square array.

square root (KS3): A number whose square is equal to a given number. Example: one square root of 25 is 5 since $5^2 = 25$. The square root of 25 is recorded as $\sqrt{25} = 5$. However, as well as a positive square root, 25 has a negative square root, since $(-5)^2 = 25$.

cube number (KS2): A number that can be expressed as the product of three equal integers. Example: 27 = 3 × 3 × 3. Consequently, 27 is a cube number; it . It is the cube of 3 or 3 cubed. This is written compactly as $27 = 3^3$, using index, or power, notation.

Division

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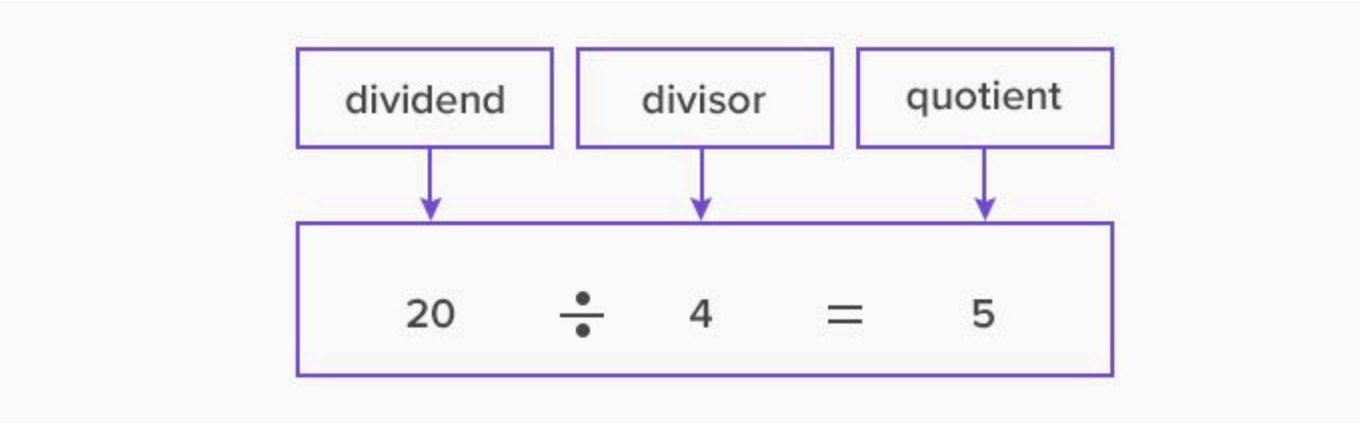
Division

divide (KS1): To carry out the operation of division.

dividend (KS1): In division, the number that is divided. E.g. in 15 ÷ 3, 15 is the dividend See also Addend, subtrahend and multiplicand.

divisor (KS2): The number by which another is divided. Example: In the calculation 30 ÷ 6 = 5, the divisor is 6. In this example, 30 is the dividend and 5 is the quotient.

quotient (KS2): The result of a division. Example: 46 ÷ 3 = 15⅓ and 15⅓ is the quotient of 46 by 3. Where the operation of division is applied to the set of integers, and the result expressed in integers, for example 46 ÷ 3 = 15 remainder 1 then 15 is the quotient of 46 by 3 and 1 is the remainder.



Division

divisibility (KS2): The property of being divisible by a given number. Example: A test of divisibility by 9 checks if a number can be divided by 9 with no remainder. January 2014 Page 43

divisible (by) (KS2): A whole number is divisible by another if there is no remainder after division and the result is a whole number. Example: 63 is divisible by 7 because 63 ÷ 7 = 9 remainder 0. However, 63 is not divisible by 8 because 63 ÷ 8 = 7.875 or 7 remainder 7.

division (KS1): An operation on numbers interpreted in a number of ways. Division can be sharing – the number to be divided is shared equally into the stated number of parts; or grouping – the number of groups of a given size is found. Division is the inverse operation to multiplication. 2. On a scale, one part. Example: Each division on a ruler might represent a millimetre.

remainder (KS2): In the context of division requiring a whole number answer (quotient), the amount remaining after the operation. Example: 29 divided by 7 = 4 remainder 1. January 2014 Page 75

repeated subtraction (KS1): The process of repeatedly subtracting the same number or amount. One model for division. Example 35 -5 - 5 - 5 - 5 - 5 - 5 - 5 = 0 so 35 ÷ 5 = 7 remainder 0.

share (equally) (KS1): Sections of this page that are currently empty will be filled over the coming weeks. One model for the process of division.

Commutative, associative and distributive laws

Commutative Laws:	$a + b = b + a$
	$a \times b = b \times a$
Associative Laws:	$(a + b) + c = a + (b + c)$
	$(a \times b) \times c = a \times (b \times c)$
Distributive Law:	$a \times (b + c) = a \times b + a \times c$

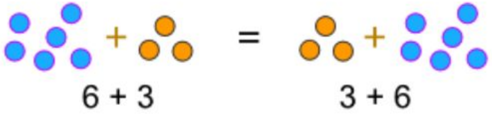
Commutative law

The "Commutative Laws" say we can **swap numbers** over and still get the same answer ...

... when we **add**:

$$a + b = b + a$$

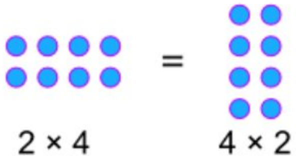
Example:



.. or when we **multiply**:

$$a \times b = b \times a$$

Example:



Commutative law

Commutative Percentages!

Because $a \times b = b \times a$ it is also true that $a\%$ of b $=$ $b\%$ of a

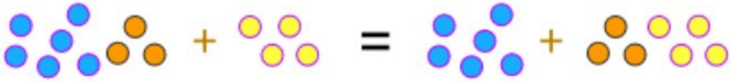
> Example: 8% of 50 = 50% of 8, which is 4

Associative law

The "Associative Laws" say that it doesn't matter how we group the numbers (i.e. which we calculate first) ...

... when we **add**:

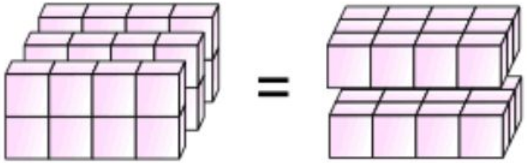
$$(a + b) + c = a + (b + c)$$



(6 + 3) + 4 = 6 + (3 + 4)

... or when we **multiply**:

$$(a \times b) \times c = a \times (b \times c)$$



(2 × 4) × 3 = 2 × (4 × 3)

Associative law

Uses:

Sometimes it is easier to add or multiply in a different order:

What is 19 + 36 + 4?

$$\begin{aligned} 19 + 36 + 4 &= 19 + \mathbf{(36 + 4)} \\ &= 19 + \mathbf{40} = 59 \end{aligned}$$

Or to rearrange a little:

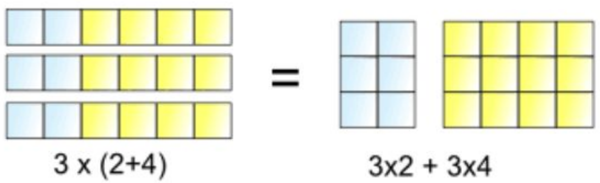
What is 2 × 16 × 5?

$$\begin{aligned} 2 \times 16 \times 5 &\mathbf{=} \mathbf{(2 \times 5)} \times 16 \\ &\mathbf{=} \mathbf{10} \times 16 = 160 \end{aligned}$$

Distributive law

The "Distributive Law" is the BEST one of all, but needs careful attention.

This is what it lets us do:



3 lots of **(2+4)** is the same as **3 lots of 2** plus **3 lots of 4**

3 times 4.

So, the **3×** can be "distributed" across the **2+4**, into **3×2** and **3×4**

And we write it like this:

$$a \times (b + c) = a \times b + a \times c$$

We get the same answer when we:

- multiply a number by a **group of numbers added together**, or
- do each **multiply** separately then **add** them

Distributive law

Uses:

Sometimes it is easier to break up a difficult multiplication:

Example: What is 6×204 ?

$$\begin{aligned} 6 \times 204 &= 6\times200 + 6\times4 \\ &= 1{,}200 + 24 \\ &= 1{,}224 \end{aligned}$$

Or to combine:

Example: What is $16 \times 6 + 16 \times 4$?

$$\begin{aligned} 16 \times 6 + 16 \times 4 &= 16 \times \mathbf{(6+4)} \\ &= 16 \times \mathbf{10} \\ &= 160 \end{aligned}$$

Distributive law

We can use it in subtraction too:

Example: $26\times3 - 24\times3$

$$
\begin{aligned}
26\times3 - 24\times3 &= \mathbf{(26 - 24)} \times 3 \\
&= 2 \times 3 \\
&= 6
\end{aligned}
$$

We could use it for a long list of additions, too:

Example: $6\times7 + 2\times7 + 3\times7 + 5\times7 + 4\times7$

$$
\begin{aligned}
&\mathbf{6}\times7 + \mathbf{2}\times7 + \mathbf{3}\times7 + \mathbf{5}\times7 + \mathbf{4}\times7 \\
&= \mathbf{(6+2+3+5+4)} \times 7 \\
&= \mathbf{20} \times 7 \\
&= \mathbf{140}
\end{aligned}
$$

However …

The Commutative Law does **not** work for subtraction or division:

Example:

- $12 / 3 =$ **4**, but
- $3 / 12 =$ **¼**

The Associative Law does **not** work for subtraction or division:

Example:

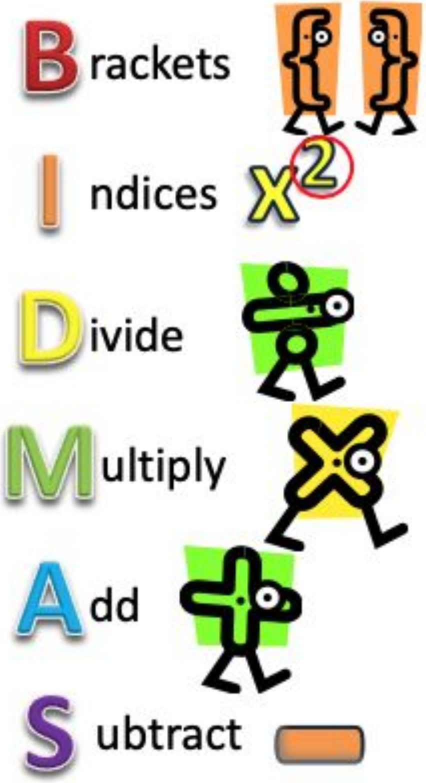
- $(9 - 4) - 3 = 5 - 3 =$ **2**, but
- $9 - (4 - 3) = 9 - 1 =$ **8**

The Distributive Law does **not** work for division:

Example:

- $24 / (4 + 8) = 24 / 12 =$ **2**, but
- $24 / 4 + 24 / 8 = 6 + 3 =$ **9**

BIDMAS



BIDMAS

order of operation (KS2) This refers to the order in which different mathematical operations are applied in a calculation. Without an agreed order an expression such as 2 + 3 × 4 could have two possible values: 5 × 4 = 20 (if the operation of addition is applied first) 2 + 12 = 14 (if the operation of multiplication is applied first) The agreed order of operations is that: • Powers or indices take precedence over multiplication or division; Multiplication or division takes precedence over addition and subtraction. If brackets are present, the operation contained therein always takes precedence over all others. This convention is often encapsulated in the mnemonic BODMAS or BIDMAS: Brackets Orders / Indices (powers) Division & Multiplication Addition & Subtraction

brackets (KS2): Symbols used to group numbers in arithmetic or letters and numbers in algebra and indicating certain operations as having priority. Example: 2 x (3 + 4) = 2 x 7 = 14 whereas 2 x 3 + 4 = 6 + 4 = 10.

priority of operations (KS2): Generally, multiplication and division are done before addition and subtraction, but this can be ambiguous, so brackets are used to indicate calculations that must be done before the remainder of the operations are carried out. See order of operation

index notation (KS2): The notation in which a product such as a × a × a × a is recorded as a^4. In this example the number 4 is called the index (plural indices) and the number represented by a is called the base. See also standard index form

Decimals


Ones
Tens
Hundreds
Decimal Point
1/10 (tenths)
1/100 (hundredths)
1/1000 (thousandths)
319.478
Values get 10x larger
Values get 10x smaller

Decimals

decimal (KS2): Relating to the base ten. Most commonly used synonymously with decimal fractions where the number of tenths, hundredth, thousandths, etc. are represented as digits following a decimal point. The decimal point is placed at the right of the ones column. Each column after the decimal point is a decimal place.

decimal fraction (KS2): Tenths, hundredths, thousandths etc represented by digits following a decimal point. Example 0.125 is equivalent to 1 /10 + 2 /100 + 5 /1000 or 1 /8 The decimal fraction representing 1 /8 is a terminating decimal fraction since it has a finite number of decimal places. Other fractions such as 1 /3 produce recurring decimal fractions. These have a digit or group of digits that is repeated indefinitely. In recording such decimal fractions a dot is written over the single digit, or the first and last digits of the group, that is repeated.

decimal system (KS2): The common system of numbering based upon powers of ten; Example: 152.34 is another way of writing 1 × 102 + 5 × 101 + 2 × 100 + 3 × 10−1 + 4 × 10−2.

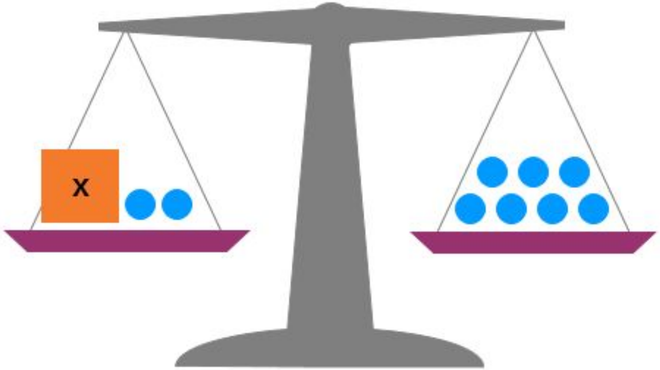
terminating decimal (KS2): A decimal fraction that has a finite number of digits. Example: 0.125 is a terminating decimal. In contrast ⅓ is a recurring decimal fraction. All terminating decimals can be expressed as fractions in which the denominator is a multiple of 2 or 5.

degree of accuracy (KS2): A measure of the precision of a calculation, or the representation of a quantity. A number may be recorded as accurate to a given number of decimal places, or rounded to the nearest integer, or to so many significant figures.

recurring decimal (KS2): A decimal fraction with an infinitely repeating digit or group of digits. Example: The fraction ⅓ is the decimal 0.33333 ..., referred to as nought point three recurring and may be written as 0.3 (with a dot over the three). Where a block of numbers is repeated indefinitely, a dot is written over the first and last digit in the block e.g. 1 /7 = 0.˙142857˙

Algebra

$x + 2 \quad = \quad 7$



Algebra

equal (KS1): Symbol: =, read as ‘is equal to’ or ‘equals’. and meaning ‘having the same value as’. Example: 7 – 2 = 4 + 1 since both expressions, 7 – 2 and 4 + 1 have the same value, 5

algebra (KS1): The part of mathematics that deals with generalised arithmetic. Letters are used to denote variables and unknown numbers and to state general properties. Example: $a(x + y) = ax + ay$ exemplifies a relationship that is true for any numbers a, x and y. Adjective: algebraic. See also equation, inequality, formula formula, identity and expression.

symbol (KS1): A letter, numeral or other mark that represents a number, an operation or another mathematical idea. Example: L (Roman symbol for fifty), > (is greater than).

rule (KS1): Generally a procedure for carrying out a process. In the context of patterns and sequences a rule, expressed in words or algebraically, summarises the pattern or sequence and can be used to generate or extend it.

Algebra

formula (KS2): An equation linking sets of physical variables. e.g. A=πr2 is the formula for the area of a circle. Plural: formulae. (the) four operations Common shorthand for the four arithmetic operations of addition, subtraction, multiplication and division.

expression (KS2): A mathematical form expressed symbolically. Examples: 7 + 3; a2 + b2

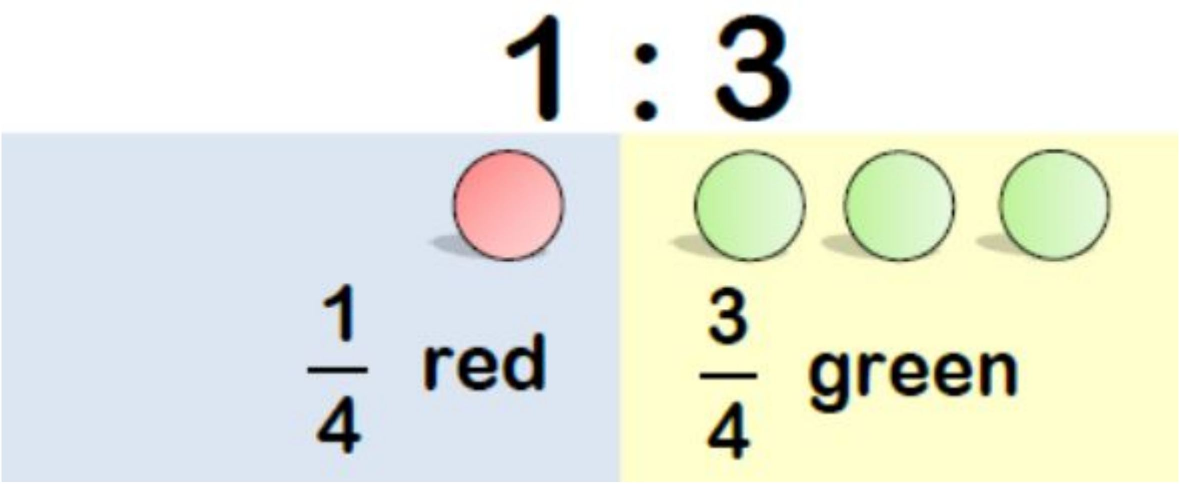
equation (KS2): A mathematical statement showing that two expressions are equal. The expressions are linked with the symbol = Examples: 7 – 2 = 4 + 1 4x = 3 x 2 – 2x + 1 = 0

equivalent expression (KS2): A numerical or algebraic expression which is the same as the original expression, but is in a different form which might be more useful as a starting point to solve a particular problem. Example: 6 + 10x is equivalent to 2(3 + 5x); 19 × 21 is equivalent to (20 – 1)(20 + 1) which is equivalent to 202 – 1 which equals 399. Equivalent expressions are identically equal to each other. Often a 3-way equals sign is used to denote 'is identically equal to'.

substitution (KS2): Numbers can be substituted into an algebraic expression in x to get a value for that expression for a given value of x. For example, when x = −2, the value of the expression 5x2 – 4x + 7 is 5(−2)2 −4(−2) + 7 = 5(4) + 8 + 7 = 35.

constant (KS2/3): A number or quantity that does not vary. Example: in the equation y = 3x + 6, the 3 and 6 are constants, where x and y are variables.

ratio and scale

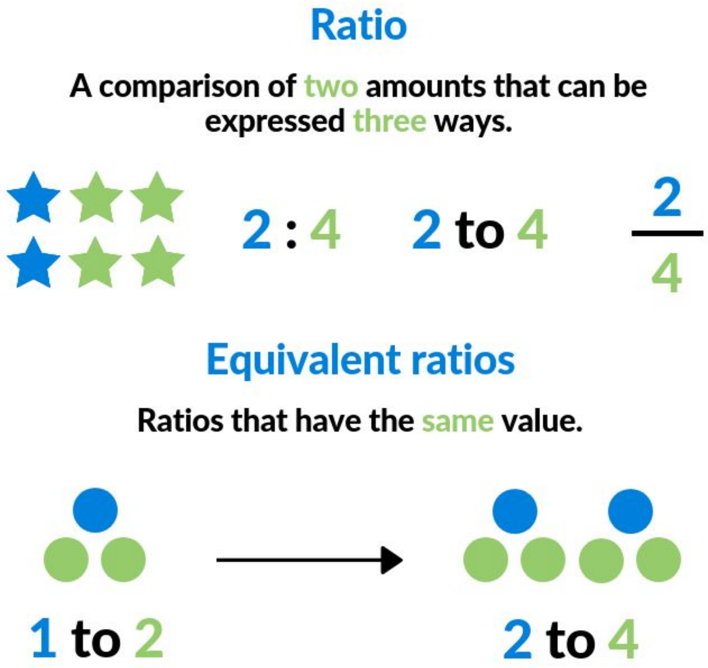


ratio

ratio (KS2): A part to part comparison. The ratio of a to b is usually written a : b. Example: In a recipe for pastry fat and flour are mixed in the ratio 1 : 2 which means that the fat used has half the mass of the flour, that is amount of fat/amount of flour = ½. Thus ratios are equivalent to particular fractional parts.

ratio notation (KS2): a : b can be changed into the unitary ratio 1 : b/a , or the unitary ratio a/b : 1. Any ratio is also unchanged if any common factors can be divided out.

correspondence problems (KS2): Correspondence problems are those in which m objects are connected to n objects (for example, 3 hats and 4 coats, how many different outfits?; 12 sweets shared equally between 4 children; 4 cakes shared equally between 8 children).

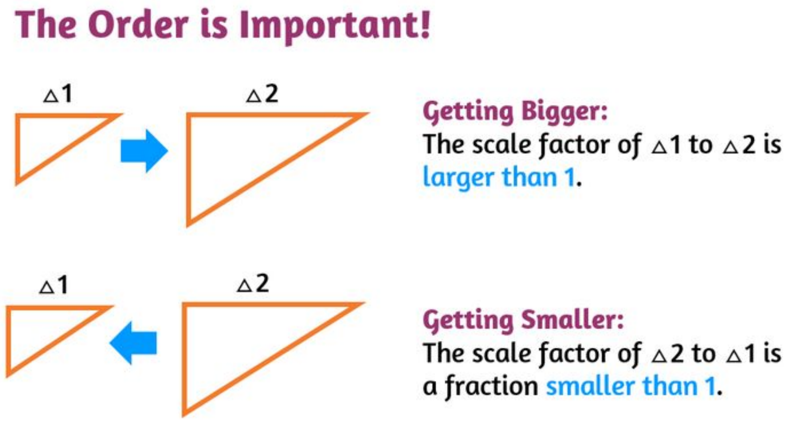


scale

Scale factor (KS2): For two similar geometric figures, the ratio of corresponding edge lengths.

scale (verb) (KS2): To enlarge or reduce a number, quantity or measurement by a given amount (called a scale factor). e.g. to have 3 times the number of people in a room than before; to find a quarter of a length of ribbon; to find 75% of a sum of money.

scale drawing or model (KS3): An accurate drawing, or model, of a representation of a physical object in which all lengths in the drawing are in the same ratio to corresponding lengths in the actual object (depending on whether the object exists in a plane or in 3 dimensions). Most maps are scaled drawings of some physical region. If the ratio of map distance to location distance is known any distance on the map can be converted to actual distance in the region represented by the map.



Fractions

fractions

fraction (KS1): The result of dividing one integer by a second integer, which must be non- zero. The dividend is the numerator and the non-zero divisor is the denominator. See also common fraction, decimal fraction, equivalent fraction, improper fraction, proper fraction, simple fraction, unit fraction and vulgar fraction.

denominator (KS2): In the notation of common fractions, the number written below the line i.e. the divisor. Example: In the fraction ⅔ the denominator is 3.

common fraction (KS1): A fraction where the numerator and denominator are both integers. Also known as simple or vulgar fraction. Contrast with a compound or complex fraction where the numerator or denominator or both contain fractions.

equivalent fractions (KS1): Fractions with the same value as another. For example: 4/8, 5/10, 8/16 are all equivalent fractions and all are equal to ½.

unit fraction (KS1): A fraction that has 1 as the numerator and whose denominator is a non-zero integer. Example: ½, ⅓

simple fraction (KS1): A fraction where the numerator and denominator are both integers. Also known as common fraction or vulgar fraction.

numerator (KS2:) In the notation of common fractions, the number written on the top – the dividend (the part that is divided). In the fraction ⅔, the numerator is 2.

fractions

vulgar fraction (KS2): A fraction in which the numerator and denominator are both integers. Also known as common fraction or simple fraction.

simplify (a fraction) (KS2): Reduce a fraction to its simplest form. See cancel (a fraction) and reduce (a fraction).

reduce (a fraction) (KS3): Divide the numerator and denominator by a common factor. To cancel a fraction. Example: divide the numerator and denominator by 5, to reduce 5 /15 to ⅓, its simplest form. January 2014 Page 74

proper fraction (KS2): A proper fraction has a numerator that is less than its denominator So ¾ is a proper fraction, whereas 4 /3 is an improper fraction (i.e. not proper).

mixed fraction (KS2): A whole number and a fractional part expressed as a common fraction. Example: 1⅓ is a mixed fraction. Also known as a mixed number.

mixed number (KS2): A whole number and a fractional part expressed as a common fraction. Example: 2 ¼ is a mixed number. Also known as a mixed fraction.

improper fraction (KS2): An improper fraction has a numerator that is greater than its denominator. Example: 9 /4 is improper and could be expressed as the mixed number 2¼

Percentages

%

Percentages

percentage (KS2): A fraction expressed as the number of parts per hundred and recorded using the notation %.

Example: One half can be expressed as 50%; the whole can be expressed as 100%

Percentage can also be interpreted as the operator ‘a number of hundredths of’. Example: 15% of Y means 15/100 × Y

Frequently, it is necessary to calculate a percentage increase, or a percentage decrease. Sometimes, given the result of an increase or decrease the original whole has to calculated.
Example 1: A salary of £24000 is increased by 5%; find the new salary. Calculation is £2400 × (1.05) = £25200 (note: 1.05 = 1 + 5/100)
Example 2: The city population of 5 500 000 decreased by 13% over the last five years so that the present population is 5500000 × (0.87) = 4 785 000 (note: 1 – 13/100 = 0.87)
Example 3: A sale item is on sale at £560 after a reduction of 20%, what was its original price? The calculation is: original price × 0.8 = £560. So, original price = £560/0.8 (since division is inverse to multiplication) = £700.

Methods, reasoning and jottings

reasoning

general statement (KS1): A statement that applies correctly to all relevant cases. e.g. the sum of two odd numbers is an even number.

g**eneralise (KS1):** To formulate a general statement or rule.

approximation (KS2): A number or result that is not exact. In a practical situation an approximation is sufficiently close to the actual number for it to be useful. Verb: approximate. Adverb: approximately. When two values are approximately equal, the sign ≈ is used.

conjecture (KS1): An educated guess (or otherwise!) of a particular result, which is as yet unverified.

counter example (KS1): Where a hypothesis or general statement is offered, an example that clearly disproves it.

relation, relationship (KS1): A common property of two or more items. An association between two or more items.

Methods and jottings

efficient methods (KS2): A means of calculation (which can be mental or written) that achieves a correct answer with as few steps as possible. In written calculations this often involves setting out calculations in a columnar layout. If a calculator is used the most efficient method uses as few key entries as possible.

mental calculation (KS1): Referring to calculations that are largely carried out mentally, but may be supported with a few simple written jottings.

inverse operations (KS1): Operations that, when they are combined, leave the entity on which they operate unchanged. Examples: addition and subtraction are inverse operations e.g. 5 + 6 – 6 = 5. Multiplication and division are inverse operations e.g. 6 × 10 ÷ 10 = 6. Squaring and taking the square root are inverse to each other: √x2 = (√x)2 = x; similarly with cube and cube root, and any integer power n and nth root. Some operations, such as reflection in the x-axis, or 'subtract from 10' are self-inverse i.e. they are inverses of themselves

missing number problems (KS1:) A problem of the type 7 = □ − 9 often used as an introduction to algebra.

notation (KS1): A convention for recording mathematical ideas. Examples: Money is recorded using decimal notation e.g. £2.50 Other examples of mathematical notation include a + a = 2a, y = f(x) and n × n × n = n3

operator (KS2): A mathematical action: In the lower key stages 'half of', 'quarter of', 'fraction of', 'percentage of ' are considered as operations. In more advanced mathematics there are very many operators that can be defined, for example a 'linear transformation' or a 'differential operator'.

compensation (in calculation) (KS1/2): A mental or written calculation strategy where one number is rounded to make the calculation easier. The calculation is then adjusted by an appropriate compensatory addition or subtraction. Examples: • 56 + 38 is treated as 56 + 40 and then 2 is subtracted to compensate. • 27 × 19 is treated as 27 × 20 and then 27 (i.e. 27 × 1) is

methods and jottings

formal written methods (KS2): Setting out working in columnar form. In multiplication, the formal methods are called short or long multiplication depending on the size of the numbers involved. Similarly, in division the formal processes are called short or long division.

columnar addition or subtraction (KS2): A formal method of setting out an addition or a subtraction in ordered columns with each column representing a decimal place value and ordered from right to left in increasing powers of 10. With addition, more than two numbers can be added together using column addition, but this extension does not work for subtraction.

Exchange (KS2): Change a number or expression for another of equal value. The process of exchange is used in some standard compact methods of calculation. Examples: ‘carrying figures’ in addition, multiplication or division; and ‘decomposition’ in subtraction.

binary operation (KS1): A rule for combining two numbers in the set to produce a third also in the set. Addition, subtraction, multiplication and division of real numbers are all binary operations.

long division (KS2): A columnar algorithm for division by more than a single digit.

long multiplication (KS2): A columnar algorithm for performing multiplication by more than a single digit.

short division (KS2) A compact written method of division. Example: 496 ÷ 11 becomes Answer : 45 1 /11

short multiplication (KS2) Essentially, simple multiplication by a one digit number, with the working set out in columns. 342 x 7 becomes Answer: 2394